Homework 5
(due Friday, October 3)

Warmup (don't turn in). 3.1: 1ac, 2ab, 3a

Turn in answers only. 3.1: 1bdef, 2cd, 3bc (in part c, you might find that the Cauchy Schwarz Inequality is useful for maximizing the directional derivative you get).

Turn in full solutions.

- Use the definition of limit to show that the function $f(x_1, x_2, x_3) = 2x_1 - 3x_2 + x_3 - 17$ is continuous at the point $(-1, 0, 3)$.
- (stronger version of 2.3.5) Suppose that $U \subset \mathbf{R}^n$ is open and $f : U \to \mathbf{R}$ is continuous. Given that $a \in U$ and $f(a) > 0$, show that there exists $\delta > 0$ such that for any $x \in B(a, \delta)$, one has $f(x) > f(a)/2$.
- 2.3: 8adfghj (use properties of limits, step by step, to find the given limit, or show that the limit does not exist by exhibiting two paths through $\mathbf{0}$ along which the function has different/no limits)
- 2.3: 12, 15a
- Prove the following version of the *Squeeze Theorem*, which we will use frequently later. Let $\mathbf{a} \in \mathbf{R}^n$ be a point and $U \subset \mathbf{R}^n$ be a neighborhood of $\mathbf{a}$. Suppose that $f : U \to \mathbf{R}^n$, $g : U \to \mathbf{R}$ are functions such that
  - $\|f(\mathbf{x})\| \leq |g(\mathbf{x})|$ for each $x \in U$; and
  - $\lim_{\mathbf{x} \to \mathbf{a}} g(x) = 0$.

  Then $\lim_{\mathbf{x} \to \mathbf{a}} f(\mathbf{x}) = \mathbf{0}$.
- 3.1: 8, 10, 11 (in 10 and 11 use the limit definition of directional derivative–it's easier)