Homework 7
(due Friday, October 17)

Warmup (don't turn in). 3.3: 1

Answers only: 3.3: 2, 3

Turn in full solutions.

- 3.2: 5, 10, 14a (Make sure to verify that the function $T(\mathbf{h}) = A\mathbf{a}\cdot\mathbf{h} + A\mathbf{h}\cdot\mathbf{a}$ is actually linear), 15

- In this problem, we find the derivative of the function $F : \mathbf{R}^{n^2} \to \mathbf{R}^{n^2}$ given by $F(A) = A^2$, where $A \in \mathbf{R}^{n^2}$ is an $n \times n$ matrix.
  (a) Recall that for any $m \times n$ matrix A and any vector $\mathbf{v} \in \mathbf{R}^n$, we have $\|A\mathbf{v}\| \leq \|A\|\,\|\mathbf{v}\|$, where $\|A\| = \sqrt{\sum a_{ij}^2}$ is the magnitude that I defined in class. Extend this fact to matrices by showing that $\|AB\| \leq \|A\|\,\|B\|$ for any $n \times p$ matrix B. It helps to think of $B = [\mathbf{b}_1 \ldots \mathbf{b}_p]$ in terms of its columns, remembering that $\|B\|^2 = \sum \|\mathbf{b}_j\|^2$.
  (b) Show that $T : \mathbf{R}^{n^2} \to \mathbf{R}^{n^2}$ given by $T(H) = AH + HA$ is a linear transformation.
  (c) Show that F is differentiable at each 'point' $A \in \mathbf{R}^{n^2}$ with derivative $DF(A) = T$.

- 3.3: 4a (do this two ways: once by using the chain rule and once by reducing the problem to finding the derivative of a function of a single variable), 8.

- Consider the function $f : \mathbf{R}^2 \to \mathbf{R}^2$ given by $f(x, y) = (x^2 - y^2, 2xy)$ (which I considered in class). Let $f^n = f \circ \cdots \circ f$ denote the n-fold composition of f with itself. Use the chain rule to find the following:
  (a) The standard matrix of $Df^n(1, 1)$ for $n = 1, 2, 3$;
  (b) Find the standard matrix of $Df^n(1, 0)$ for *any* positive integer n.