1. Find all solutions to $\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

(a) $\begin{pmatrix} 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (e) $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$

2. The number of pivots in the reduced row echelon form of $A = \begin{pmatrix} 3 & -9 & 12 & -9 \\ 0 & 2 & -4 & 4 \\ 0 & 3 & -6 & 6 \end{pmatrix}$ is

(a) 2 (b) 1 (c) 3 (d) 4 (e) 0

3. The system $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 3 & 6 \\ 1 & 4 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \\ 13 \end{pmatrix}$ has

(a) infinitely many solutions (b) exactly one solution

(c) no solution (d) two solutions

(e) three solutions

4. Find the value of k such that

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 15 \\ k \end{pmatrix}$$

has at least one solution.

(a) 24 (b) 18 (c) 9 (d) 7 (e) 32

5. The matrix product $\begin{pmatrix} 2 & 2 & 2 \\ 3 & 3 & 9 \\ -1 & 7 & 8 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ is equal to

(a) does not make sense (b) $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

(d) $\begin{pmatrix} 2 \\ 9 \\ 7 \end{pmatrix}$ (e) 0

6. Find a *non-zero* solution to $\begin{pmatrix} -6 & -2 \\ 21 & 7 \\ -9 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(a) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} -2 \\ 7 \\ -3 \end{pmatrix}$

7. Find all values of h for which the vectors $\{\begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ 10 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ -7 \\ h \end{pmatrix}\}$ are linearly independent.

(a) none (b) 1 (c) 2 (d) 3 (e) 0

8. The product $(0, 0, 1) \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(a) h (b) f (c) b (d) d (e) k

9. Let $A = \begin{pmatrix} 1 & 5 & 4 & 3 & 2 \\ 1 & 6 & 6 & 6 & 6 \\ 1 & 7 & 8 & 10 & 12 \\ 1 & 6 & 6 & 7 & 8 \end{pmatrix}$. The dimension of the column space $Col(A)$ is equal to

(a) 3 (b) 4 (c) 2 (d) 1 (e) 5

10. Let $T(\vec{x}) = A\vec{x}$ be a rotation of $\frac{\pi}{4}$ in counter-clock-wise direction on the Euclidean plane R^2, where A is a 2×2 matrix. Then the matrix A is equal to

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (e) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

11. The inverse of $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$ is

(a) $\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$ (c) $\begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$ (e) $\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$

12. Let $Nul(A)$ be the null space of A. Suppose that $A = \begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{pmatrix}$. Find the $\dim[Nul(A)]$.

(a) 2 (b) 3 (c) 1 (d) 4 (e) 0

13. Find the reduced echelon form of $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{pmatrix}$.

14. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{pmatrix}$. Find the inverse of A.

15. Let $A = \begin{pmatrix} 3 & 1 \\ 9 & 3 \end{pmatrix}$. Construct a *non-zero* 2×2 matrix B such that AB is the zero matrix.