1. Let $A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 3 & 2 \\ 4 & 5 & 1 \end{pmatrix}$. The rank of A is

(a) 4 (b) 3 (c) 1 (d) 2 (e) 0

2. Let $\mathbf{P}_2 = \{a_0 + a_1 t + a_2 t^2\}$ where $\{a_0, a_1, a_2\}$ range over all real numbers, and let $T : \mathbf{P}_2 \to \mathbf{P}_2$ be a linear transformation defined by

$$T(a_0 + a_1 t + a_2 t^2) = a_1 + 9a_2 t.$$

If $T(p(t)) = \lambda p(t)$ for some non-zero polynomial $p(t)$ in $\mathbf{P}_2$ and some real number λ, then $p(t)$ is called an eigenvector of T corresponding to λ. The linear transformation T has an eigenvector:

(a) t (b) $t + 9t^2$ (c) 100 (d) t^2 (e) 0

3. Let $\{\lambda_1, \lambda_2\}$ be two eigenvalues of $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$. Then the product of two eigenvalues, $\lambda_1 \lambda_2$, is equal to

(a) -28 (b) 4 (c) 7 (d) 3 (e) 28

4. Let $\mathbf{b}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find $[\mathbf{x}]_{\mathbf{B}}$.

(a) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ (e) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

5. The matrix $A = \begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}$ has a complex eigenvector:

(a) $\begin{pmatrix} 3 \\ 4i \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ i \end{pmatrix}$ (c) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} 4 \\ 3i \end{pmatrix}$ (e) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

6. Let $A = \begin{pmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{pmatrix}$. The eigenvalues of A are

(a) $0, 1, 2$ (b) $1, -2, -2$ (c) $1, 2, 3$ (d) $1, \pm 2$ (e) $0, 1, -2$

7. Let S be the parallelogram determined by the vectors $\mathbf{b}_1 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and let $A = \begin{pmatrix} -2 & 99 \\ 0 & 1 \end{pmatrix}$. Compute the area of the images of S under the mapping $\mathbf{v} \longmapsto A\mathbf{v}$.

(a) 2 (b) 99 (c) 3 (d) 5 (e) -2

8. Let $A = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}$. The complex eigenvalues of A are

(a) $3 \pm i$ (b) $1 \pm 3i$ (c) $\pm 3i$ (d) 4 (e) $\pm i$

9. For what value(s) of h will $\mathbf{y}$ be in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, if $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 6 \\ 4 \\ h \end{pmatrix}$.

(a) 2 (b) 4 (c) 3 (d) 6 (e) 10

10. Let $\mathbf{b}_1 = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$, $\mathbf{c}_1 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$, $\mathbf{c}_2 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$, $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$. If $P = [[\mathbf{b}_1]_{\mathbf{C}}, [\mathbf{b}_2]_{\mathbf{C}}]$ is the change-of-coordinates matrix from $\mathbf{B}$ to $\mathbf{C}$, then find P.

(a) $\begin{pmatrix} -9 & -5 \\ 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 3 \\ -4 & -5 \end{pmatrix}$ (c) $\begin{pmatrix} 6 & 4 \\ -5 & -3 \end{pmatrix}$

(d) $\frac{1}{4}\begin{pmatrix} 1 & -5 \\ 1 & 9 \end{pmatrix}$ (e) $\frac{1}{7}\begin{pmatrix} -5 & 3 \\ 4 & 1 \end{pmatrix}$

11. Find a matrix A such that $W = Col(A)$ where $W = \{ \begin{pmatrix} 9a - 8b \\ a + 2b \\ -5a \end{pmatrix} \}$ and $\{a, b\}$ range over all real numbers.

(a) $\begin{pmatrix} 9 & 1 & -5 \\ -8 & 2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 9 & -8 \\ 1 & 2 \\ -5 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 9 & -8 \\ 1 & 2 \\ 0 & -5 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

12. Let $S = \{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \}$. Then the subset S

(a) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is orthogonal to $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

(b) spans R^3

(c) a linearly dependent subset

(d) a linearly independent subset

(e) a basis of R^3

13. Let $A = \begin{pmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{pmatrix}$. Use Cramer's rule to solve $A\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

14. Let $P_2 = \{a_0 + a_1 t + a_2 t^2\}$ where $\{a_0, a_1, a_2\}$ range over all real numbers, and let $T : P_2 \to P_1$ be a linear transformation given by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$. Suppose that $\mathbf{B} = \{1, t, t^2\}$ is a basis of P_2 and $\mathbf{C} = \{1, t\}$ is a basis of P_1.

(1) Find a matrix A such that $[T\mathbf{v}]_{\mathbf{C}} = A[\mathbf{v}]_{\mathbf{B}}$;

(2) Find $Nul(A)$ and $Col(A)$.

15. Let $A = \frac{1}{5}\begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$. Find $\lim_{k\to\infty} A^k$. (Hint: Find the Diagonalization D of A and use the formula $A^k = PD^kP^{-1}$).