1. Let $A = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$, $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ and let $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$. Find the angle between $A\vec{u}$ and $A\mathbf{v}$.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0 (e) $\frac{\pi}{6}$

2. Let $\mathbf{y} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ and $\vec{u} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Find orthogonal projection of $\vec{y}$ onto $\vec{u}$.

(a) $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ (b) $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

3. Let $\mathbf{y} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$, $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ and $W = Span\{\mathbf{v}_1, \mathbf{v}_2\}$. Use the fact that $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal to compute $Proj_W \mathbf{y}$.

(a) $\begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$ (d) $\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

4. Find the distance between $\mathbf{y}$ and W, where $\mathbf{y} = \begin{pmatrix} 3 \\ -1 \\ 1 \\ 13 \end{pmatrix}$, $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix}$

and $W = Span\{\mathbf{v}_1, \mathbf{v}_2\}$.

(a) 8 (b) 0 (c) 1 (d) 3 (e) 13

5. Find a least-squares solution of inconsistent system $A\mathbf{x} = \mathbf{b}$ for $A = \begin{pmatrix} 0 & 4 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$ and

$\mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$.

(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

6. Find $\vec{u}_3$ so that the subset $\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}, \vec{u}_3\}$ becomes an orthogonal basis of $W =$ $Span\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\3\\1\\7\end{pmatrix}\}$

(a) $\begin{pmatrix}-3\\1\\-1\\3\end{pmatrix}$ (b) $\begin{pmatrix}1\\3\\1\\7\end{pmatrix}$ (c) $\begin{pmatrix}4\\2\\2\\4\end{pmatrix}$ (d) $\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$ (e) $\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

7. Find an orthonormal basis of the subspace $W = Span\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\9\\9\\1\end{pmatrix}\}$.

(a) $\{\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \frac{1}{2}\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\}$ (b) $\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\}$ (c) $\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\9\\9\\1\end{pmatrix}\}$

(d) $\{\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\9\\9\\1\end{pmatrix}\}$ (e) $\{\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\9\\-9\\0\end{pmatrix}\}$

8. Let $A = \frac{1}{7}\begin{pmatrix} 2 & 6 & 3 \\ 3 & 2 & -6 \\ 6 & -3 & 2 \end{pmatrix}$. Find A^{-1}.

(a) $\frac{1}{7}\begin{pmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{pmatrix}$ (b) A^2 (c) A^3

(d) 0 (e) $\begin{pmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{pmatrix}$

9. Solve the initial value problem of $ty' + 2y = 4t$ with the initial condition $y(1) = 3$

(a) $t^2 + \frac{2}{t^2}$ (b) $t^2 + \frac{1}{t^2}$ (c) $t^2 - \frac{1}{t^2}$ (d) $2t^2 + \frac{1}{t^2}$ (e) $t^2 - \frac{2}{t^2}$

10. Solve equation $y' = 9.8 - \frac{y}{5}$ with initial condition $y(0) = 50$.

(a) $49 + e^{-\frac{t}{5}}$ (b) $1 + 49e^{-\frac{t}{5}}$ (c) 50 (d) 9.8 (e) 49

11. Find all solutions to the separable equation $y' = \frac{x^2}{y(1+x^3)}$.

(a) $3y^2 - 2\ln|1+x^3| = c$ (b) $3y^2 - \ln|1+x^3| = c$ (c) $y^2 - 2\ln|1+x^3| = c$

(d) 0 (e) $2y^2 - 3\ln|1+x^3| = c$

12. Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously. Find the return rate that must be achieved if the initial investment is to double in 10 years.

(a) $\dfrac{\ln 2}{10}$ (b) $\dfrac{\ln 10}{2}$ (c) 10% (d) 20% (e) 2

13. Find the solution of $\dfrac{dy}{dt} = \dfrac{1}{2}(1-y)y$ with $y(0) = 4$ and find $\lim_{t\to\infty} y(t)$.

14. Find an integrating factor for the equation $(3xy + y + 1)dx + (x^2 + xy)dy = 0$ and then solve the equation.

15. Find $\min_{\mathbf{x}}\{\|A\mathbf{x}-\mathbf{b}\|\}$, where $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$. (Hint: the least squares solution $\mathbf{x}^*$ is given by $(A^TA)^{-1}A^T\mathbf{b}$).