1. Let $y_1(t)$ and $y_2(t)$ are two fundamental solutions $y'' + y' + \frac{\sin t}{t} y = 0$ with initial conditions $y_1(0) = 1, y_1'(0) = 0$ and $y_2(0) = 0, y_2'(0) = 1$. Then the Wronskian $W(t) = [y_1(t)y_2'(t) - y_1'(t)y_2(t)]$ is equal to

(a) $\frac{\sin t}{t}$. (b) e^t. (c) $\frac{1}{t}$. (d) e^{-t}. (e) $\sin t$.

2. Let $Y(t) = A_0t^2 + A_1t + A_2$ be a solution to $y'' + 4y = 4t^2$ where $\{A_0, A_1, A_2\}$ are constant numbers. Then A_2 is equal to

(a) -1 (b) 4 (c) 1 (d) 0 (e) $-\frac{1}{2}$

3. The linear system $\begin{pmatrix} 1 & 5 & -3 \\ 1 & 4 & -1 \\ 2 & 7 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \\ h \end{pmatrix}$ has a solution if and only if $h =$

(a) 2 (b) 1 (c) 5 (d) 3 (e) -5

4. Suppose that $Y(t) = At^s e^{-t} + B$ is a solution to $y'' - 3y' - 4y = -5e^{-t} - 4$, where $\{A, B, s\}$ are constant numbers. Then A is equal to

(a) $-\dfrac{2}{5}$ (b) 4 (c) 1 (d) -4 (e) 1

5. Find the adjoint $adj(A)$ of $A = \begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}$.

(a) $\begin{pmatrix} 7 & 4 \\ 2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -7 & -4 \\ -2 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 7 & -4 \\ -2 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 7 & -2 \\ -4 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} -7 & 4 \\ 2 & -1 \end{pmatrix}$

6. The reduced row echelon form of $\begin{pmatrix} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{pmatrix}$ is equal to

(a) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

7. Find the integrating factor μ for $dx + (\frac{x}{y} - \sin y + y^2)dy = 0$.

(a) y (b) $\sin y$ (c) 1 (d) y^2 (e) x

8. Let $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = proj_V \vec{u}$ where $\vec{u} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 7 \end{pmatrix}$ and $V = Span\{\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}\}$. Then x_1 is equal to

(a) 2 (b) 4 (c) 1 (d) 0 (e) 3

9. Which of the following sets is an orthonormal basis of R^2?

(a) $\{\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ 3 \end{pmatrix}\}$

(b) $\{\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \frac{1}{5}\begin{pmatrix} -4 \\ 3 \end{pmatrix}\}$

(c) $\{\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \frac{1}{5}\begin{pmatrix} -4 \\ 3 \end{pmatrix}, 0\}$

(d) $\{\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}\}$

(e) $\{\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \frac{1}{5}\begin{pmatrix} -4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\}$

10. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$. Then b_{11} is equal to

(a) 1 (b) −2 (c) 10 (d) −6 (e) 5

11. Let $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be a solution to $\begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 4 & -5 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ 9 \end{pmatrix}$. Then x_1 is equal to

(a) 3 (b) 16 (c) 8 (d) 9 (e) 29

12. Use the method of reduction of order to find a second solution $y_2 = v(t)y_1(t)$ of the given differential equation $t^2y'' + 2ty' - 2y = 0$ where $y_1(t) = t$. Then $v(t)$ is equal to

(a) $\frac{4}{t}$ (b) t (c) t^{-3} (d) 1 (e) t^{-2}

13. Let r_1 and r_2 be two roots of the characteristic equation for $y'' + 100y = 0$. Then r_1 and r_2 are

(a) $\pm 10\sqrt{-1}$ (b) $0, 10$ (c) $-100, 0$

(d) ± 10 (e) $-10 \pm 10\sqrt{-1}$

14. If $\det A = 2$ where A is a 4×4 matrix, then $\det(-2A)$ is

(a) 32 (b) -4 (c) -32 (d) 16 (e) -16

15. The following two solutions form a fundamental set of solutions of linear homogeneous differential equation $2t^2y'' + 3ty' - y = 0$.

(a) $t^{\frac{3}{2}}, t$ (b) t, t^{-1} (c) $t, 1$ (d) $t^{\frac{1}{2}}, t^{-1}$ (e) $t^{\frac{1}{2}}, 0$

16. If $\mathbf{B} = \{(1 \quad 0), (1 \quad 2)\}$ and $\vec{x} = (1 \quad 6)$, then $[\vec{x}]_{\mathbf{B}}$ is equal to

(a) $(1 \quad 6)$ (b) $(1 \quad 0)$ (c) $(3 \quad 2)$ (d) $(-2 \quad 3)$ (e) $(1 \quad 2)$

17. The eigenvalues of $A = \begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$ are

(a) $-3, -5, -3$ (b) $1, -5, 0$ (c) $1, 3, 3$ (d) $1, 3, 5$ (e) $1, -2, -2$

18. Let $y(t)$ be the unique solution to the initial value problem $y'' - y = 0$, $y(0) = 2$, $y'(0) = 0$. Then $y(1)$ is equal to

(a) $2e$ (b) 2 (c) $2e^{-1}$ (d) $e + e^{-1}$ (e) $2e - 2$

19. Let $y(t)$ be the unique solution to $y' + \frac{2}{t}y = 4t$ with initial condition $y(1) = 3$. Then $y(2)$ is equal to

(a) 8 (b) $\ln 2 + 2$ (c) $4 + \frac{1}{2}$ (d) $e^4 + 2$ (e) $8 + \frac{1}{4}$

20. Let $Y(t) = v_1(t)\cos 3t + v_2(t)\sin 3t$ be a solution to $y'' + 9y = \frac{1}{\sin 3t}$. Then $v_2(t)$ is equal to

(a) $\frac{1}{\sin 3t}$ (b) $\frac{t}{3}$ (c) $\cos 3t$

(d) $\frac{1}{9}\ln|\sin 3t|$ (e) $\frac{1}{3}\ln|\sin 3t|$

21. If $y' = 2y^{100}(3 - y)$ and $y(0) = 5$, then find $\lim_{t\to\infty} y(t)$. (Hint: This is an autonomous equation. You can find the answer by studying graphs of the solution).

(a) 2 (b) 3 (c) 1 (d) 0 (e) 5

22. Let $y(t)$ be the unique solution to the initial value problem $y'' + 2y' + y = 0$, $y(0) = 1$, $y'(0) = 0$. Then $y(1)$ is equal to

(a) $2e$ (b) $2e^{-1}$ (c) $e + e^{-1}$ (d) 1 (e) 0

23. Let $y(t)$ be the unique solution to the equation $y' = y^2$ with $y(0) = -1$. Then $y(1)$ is equal to

(a) 0 (b) $\frac{-1}{2}$ (c) -4 (d) -1 (e) -3

24. The determinant of $\begin{pmatrix} 1 & 5 & 0 \\ 2 & 4 & 1 \\ 0 & -2 & 0 \end{pmatrix}$ is equal to

(a) 1 (b) 2 (c) -2 (d) 0 (e) 5