Review sheet for the midterm

Standard disclaimer: The following represents a sincere attempt to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention.

Format: The exam will take place Wedensday, March 18 from 7-9 PM in Hayes-Healy 129. There will be no take home portion. However, you *will* need to turn in Homework 7 the day of the exam. I'll take any math questions you might have concerning the exam on Monday 3/16 or Wednesday 3/18.

The basic format of the exam will be similar to the (in class portion of the) midterm and final from last semester—a section of statements, a section of true false, and then several partial credit problems. I'll likely lean harder on computational problems this time around, since we've learned a fair number of computational methods in the first half of this term. I might also include problems of the type "give an example illustrating xxx" (kind of a mix of a true/false and statement problem).

Things to know:

definitions and statements. You should be able to give valid definitions of *isometry*, *adjoint operator*, *normal operator*, *self-adjoint operator*, *spectral theorems* for normal and self-adjoint operators, *positive operator*, *singular value*, *polar decomposition* of a linear transformation, *operator norm* of a linear transformation, *order of a differential equation*, *linear ode*, *autonomous ode*, *equilibrium point* of an autonomous ode, *existence and uniqueness theorem* for first order ordinary differential equations, *independent collection of subspaces* of a vector space, *generalized eigenvector* for a linear transformation, *exponential of a matrix*.

knowledge useful for answering questions. Recognizing isometries, normal and (positive) self-adjoint operators (it might help to know a couple of simple examples of each of these). Understanding what singular values "mean" for a linear transformation. Understanding how a first order ODE (or first order autonomous system of two ODEs) can be 'visualized' as a vector field in $\mathbf{R}^2$. Describing asymptotic behavior of solutions to first order autonomous ODEs and classifying equilibrium solutions as stable, unstable, or neither. Determining whether a given set of functions is dependent or independent.

computational skills. Finding the adjoint of a given linear operator. Finding 'the' square root of a positive operator, polar decomposition of a linear operator, the singular values, the operator norm and the singular value decomposition of a linear transformation. Solving first order separable and linear ODEs and initial value problems, solving (homogeneous and inhomogeneous) higher order linear ODEs with constant coefficients, finding the exponential of a matrix and using this to solve a first order linear system of ODEs.