Info sheet for the final

Standard disclaimer: The following represents a sincere attempt to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention.

Format: The exam will take place Tuesday, May 8 from 8-10 AM in Hayes-Healy 125 (our usual classroom). There will be no take home portion. I will hold office hours on Monday May 7 from 3:30-5:30 PM in case you'd like to go over anything ahead of the final.

The basic format of the exam is likely to be similar to the (in class portion of the) midterm and final from last semester—a section of statements, a section of true false (or maybe just requests for examples of various things), and then several partial credit problems. My main interest is to know how well you've mastered the basics of linear algebra and ordinary differential equations over the last year. Of course, I'm as likely as not to include a ringer or two at the end of the exam.

Content: Anything on the review sheets for this semester's midterm and last semester's midterm and final will be fair game. Beyond these, I might also ask questions about the ODE material (systems of ODEs, matrix exponential, etc) we covered after midterm this semester. The only things I expect you to know about concerning the canonical forms material are invariant subspaces and, in particular, cyclic subspaces associated to a linear operator; the Cayley-Hamilton theorem; and the SN decomposition of an operator (especially how it allows one to exponentiate a non-diagonalizable matrix). I will not ask about quotient spaces, direct sums, polynomial ideals, etc. I will not ask about minimal polynomials, the primary or cyclic decomposition theorems or about Jordan canonical forms.

You may assume throughout the exam that the scalar field is $\mathbf{R}$, except in definitions and statements (e.g. the SN decomposition theorem, the spectral theorem for normal operators) that require the vector spaces at issue to be complex.)

To spell it out a little further, here are some particular things we've covered since midterm that I might ask about on the final:

- **definitions and statements.:** You should be able to give valid definitions of *operator norm*, *exponential of a matrix*, *invariant subspace*. You should be able to state the *Cayley-Hamilton Theorem* and the *SN decomposition theorem*.
- **knowledge useful for answering questions.:** Understand the relationship between a 2×2 constant coefficient homogeneous linear system of ODEs $\mathbf{y}' = A\mathbf{y}$ and its direction field. Be able to determine whether a given subspace is invariant for some operator.
- **computational skills.:** Solving constant coefficient linear systems of ODEs, homogeneous and inhomogeneous; exponentiate a matrix; compute an operator norm; given an operator $T : V \to V$ and a vector $\mathbf{v} \in V$, know how to find the cyclic subspace $H_{\mathbf{v}}$ and the characteristic polynomial $p_{\mathbf{v}}$ of the restriction $T|_{H_{\mathbf{v}}}$.