Homework 2
(due Friday, January 31)

From Beals:

- 10C: 1, 2
- 10D: 1 , 3, 4, 5

Hints: Note that Beals provides hints for many problems in the back of his book. I think 10D, #5 is somewhat tricky, so I want to present Beals' hint for that one here. It suffices to argue as follows:

Claim: *if there is a subsequence $(A_{n_j}) \subset (A_n)$ converging to some set $A \subset \mathbf{R}$, then the full sequence (A_n) converges to A.*

Note that '(A_n) converges to A' means that $\lim d(A_n, A) = 0$. Since (A_n) is a Cauchy sequence, one can extract a subsequence $(A_{n_j}) \subset (A_n)$ such that $d(A_{n_j}, A_{n_{j+1}}) \leq 2^j$ for all $j \in \mathbf{N}$ (explain why!). Then for this subsequence one can show:

Claim: $\lim_{j\to\infty} A_{n_j} = \liminf_{j\to\infty} A_{n_j}$