Homework 10
(due Friday, April 17)

Warmup Problems (Do not turn in).
8.2: 4ce, 8abeg, 11abdfg, 12

Turn in answers only.
8.2: 4abd, 6, 8cf (justify your answers), 11ceh

Turn in full solutions.
8.2: 2, 9, 10 (you may assume that λ is C^1 here), 13, 14

Problem 1. Do 8.2.17b in Shifrin, but use the following approach: since any linear n-form on $\mathbf{R}^n$ is a constant multiple of $dx_1 \wedge \cdots \wedge dx_n$, we know that

$$\omega_1 \wedge \cdots \wedge \omega_n = C\, dx_1 \wedge \cdots \wedge dx_n.$$

for a constant $C = C(\omega_1, \ldots, \omega_n)$ depending on the given forms $\omega_j = \sum_{i=1}^n a_{ij} dx_i$. That is C is a function of the matrix $A = (a_{ij})$ whose jth column consists of the coefficients of ω_j. Therefore, it suffices to argue that (as a function of A) $C(A)$ satisfies all (three) conditions in the definition of determinant.

Problem 2. Let $F : \mathbf{R}^n \to \mathbf{R}^n$ be a C^1 function, and write $\mathbf{y} = F(\mathbf{x})$ (i.e. $\mathbf{x}$ denotes points in the source and $\mathbf{y}$ in the target. Show that

$$F^*(dy_1 \wedge \cdots \wedge dy_n) = (\det DF)\, dx_1 \wedge \cdots \wedge dx_n.$$