

Homework 7

(due Friday, March 20)

For the sake of staying in good technical shape, I advise/encourage you to do as many integrals as you can on this assignment by hand and rely on e.g. Mathematica only to check your answers.

Turn in answers only.

From Jones, Chapter 2: 2-13, 2-15

8.3: 1acef, 2ac (Don't forget to sketch the curves and explain 'coincidences' among the answers)

Problem 1. Compute the arc-length integral $\int_{\gamma} f ds$ for

(a) $\gamma : [1, 3] \rightarrow \mathbf{R}^3$ given by $\gamma(t) = (t, 3t, 2t)$ and $f : \mathbf{R}^3 \rightarrow \mathbf{R}$ given by $f(x, y, z) = yz$.

(b) $\gamma : [0, 2\pi] \rightarrow \mathbf{R}^3$ given by $\gamma(t) = (\sin t, \cos t, t)$ and $f : \mathbf{R}^3 \rightarrow \mathbf{R}$ given by $x + y + z$.

Turn in full solutions.

From the 1st midterm: 2 (find and give counterexamples to all 4 false statements), 7ab (don't use cylindrical or spherical coordinates in a)

8.3: 3abd, 4 (you can use Mathematica on this one)

From Jones, Chapter 11: 11-1

Problem 2. Find a parametrization for the path from $(0, 0, 0)$ to $(1, 1, \sqrt{2})$ along the intersection between the surfaces $\{x^2 + y^2 = z^2\}$ and $\{y^2 = x\}$.

Problem 3. Compute the average distance to the origin among points on the circle in \mathbf{R}^2 with center $(1, 0)$ and radius 1.

Problem 4. The graph $\{(x, f(x)) \in \mathbf{R}^2 : x \in [a, b]\}$ of a C^1 function $f : [a, b] \rightarrow \mathbf{R}$ is a curve which may be parametrized by its x -coordinate, i.e. by $\gamma : [a, b] \rightarrow \mathbf{R}^2$ given by $\gamma(t) = (t, f(t))$. Use this parametrization to obtain the following formula for the length of the graph of f :

$$\int_a^b \sqrt{1 + f'(x)^2} dx.$$