Homework 9

(due Friday, April 10)

Warmup Problems (Do not turn in).

1.5: 5a, 6a, 7a

Turn in answers only.

1.5: 5b, 6bc

Turn in full solutions.

1.5: 7b, 13 (assume that the origin is one of the vertices of the parallelogram), 14, 15

Problem 1. Find the area of the helicoid surface S, parametrized by $F : [-1,1] \times [0, 2\pi] \to \mathbf{R}^3$, $F(s,t) = (s\cos t, s\sin t, t)$. While it's not strictly necessary, you owe it to yourself (by using Mathematica to plot it, or just thinking it through) to see what this surface looks like.

Problem 2. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a C^1 function and $\Omega \subset \mathbf{R}^2$ be a region. Let S be the portion of the graph of f lying over Ω. That is,

$$S = \{x, y, z \in \mathbf{R}^3 : (x,y) \in \Omega \text{ and } z = f(x,y)\}.$$

(a) Show (using our definition of surface area) that the area of S is given by

$$\int_\Omega \sqrt{1 + f_x^2 + f_y^2}\, dV.$$

(Here $f_x := \frac{\partial f}{\partial x}$, etc.)

(b) Jones 11.12.

(c) Jones 11.13.

Problem 3. Let $[a,b] \subset \mathbf{R}$ be a closed interval and $f : [a,b] \to \mathbf{R}$ be a non-negative C^1 function. Let S be the surface in $\mathbf{R}^3$ obtained by revolving the graph of f about $[a,b]$. That is,

$$S = \{(x,y,z) \in \mathbf{R}^3 : z \in [a,b] \text{ and } \|(x,y)\| = f(z)\}.$$

(a) Show that the area of S is given by

$$2\pi \int_a^b f(z)\sqrt{1 + f'(z)^2}\, dz.$$

Then use this formula to find the surface area of

(b) the sphere $\partial B(0,R) \subset \mathbf{R}^3$; and

(c) a right circular cone with height h and base radius r. Do not include the area of the base here.