Synthesis of 3R Linkage

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The synthesis of a three-revolute (3R) linkage, schematically shown in Fig. 1 from [4, Fig. 2], for body guidance was formulated as an open problem in 2002 in [2] which, along with [3, 4], solved special cases. In particular, the general problem of synthesizing 3R linkages that achieve the maximum number of 5 given poses remained open for 15 years when is was solved by `Bertini` in [1] yielding 456 such linkages. In this example, we utilize the solution to the general synthesis problem to demonstrate using a paramater homotopy in `Bertini` to solve the instance considered in [4] by tracking exactly 456 paths.

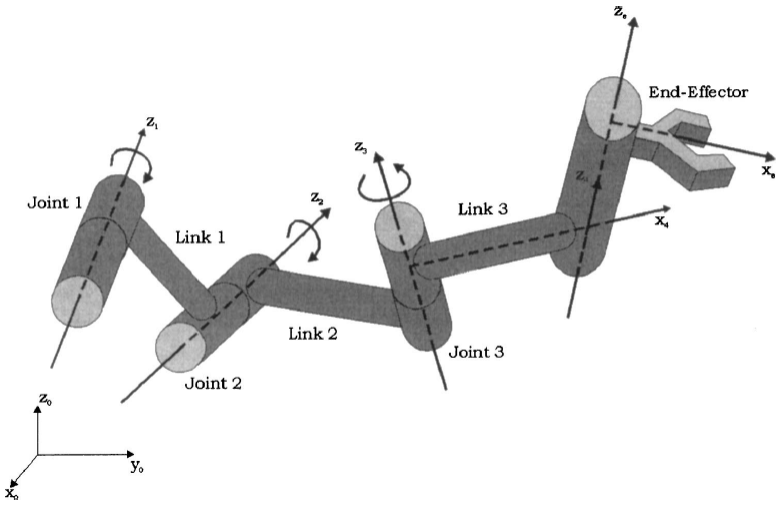

FIGURE 1. Schematic of 3R linkage from [4, Fig. 2]

The parameters for the synthesis problem correspond with the location and orientation (written in terms of quaternions) of the five given poses, which are deformed from the general case (listed in `start_parameters`) to the instance considered in [4] (listed in `final_parameters`). As described in [1, § 5.1], the interval analysis approach used in [4] used 5 days of computations on a cluster to compute 26 real solutions. Executing `Synthesis.sh` calls `Bertini` using the input file `inputSynthesis` which describes a parameter homotopy and uses the 456 solutions to the general synthesis problem listed in `startSynthesis` as the start points. This computation finds *all* solutions in a few minutes on a single core (a few seconds on a cluster in parallel). In particular, this computation shows that there are actually 28 real solutions to the 3R synthesis problem considered in [4].

REFERENCES

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