

Illustrative Example Sharpened to 400 Digits

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To demonstrate the ability of `Bertini` to compute solutions to arbitrary accuracy, we utilize the sharpening module on the illustrative quintic polynomial equation, namely:

$$(1) \quad f(x) = x^5 - x + 1 = 0$$

to refine the solutions to 400 digits using Newton's method. Using `IllustrativeSharpen.sh`, `Bertini` is called with input file `inputIllustrativeSharpen`. In our test, `Bertini` used 1344-bit floating-point arithmetic to approximate the 5 solutions to 400 digits of accuracy. The unique real solution is listed in the the output file `real-finite-solutions` and partially presented here:

```
-0.11673039782614...[390 digits]...72e1 0.13021344588521...[390 digits]...80e-404
```

which shows that the imaginary part of the numerical approximation is on the order of 10^{-404} . One can prove that the exact solution is real using [1].

REFERENCES

- [1] J.D. Hauenstein and F. Sottile. Algorithm 921: `alphaCertified`: certifying solutions to polynomial systems. *ACM Trans. Math. Software*, 38(4), 28, 2012.