Number of Real Plane Conics

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Consider the enumerative geometry problem of counting plane conics in $\mathbb{C}^3$ that pass through the origin and intersect 6 given lines. Figure 1 illustrates one such plane conic for 6 given lines. In fact, when the lines are general, the total number of such plane conics is 18. Let a *real line* and *real plane conic* be a line and plane conic in $\mathbb{C}^3$, respectively, that can be defined by equations with real coefficients. With this, consider the following real enumerative geometry problem.

Problem 1. *Is it possible to construct* 6 *real lines such that there are exactly* 18 *nonreal plane conics, i.e., no real plane conics, that pass through the origin and intersect these lines?*

An experiment involving 1,000,000 randomly selected instances was performed in [2, § 5.3] (see also [1, § 4.2]) and was unable to find 6 real lines that satisfy Problem 1.

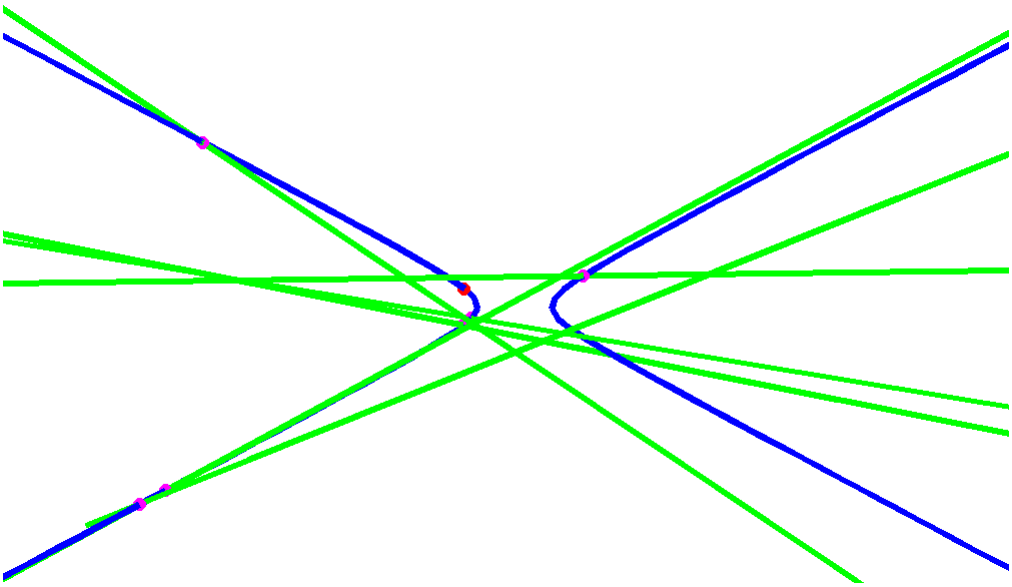

FIGURE 1. Plot of a plane conic (blue) passing through the origin (red) and intersecting 6 lines (green).

REFERENCES

[1] Z.A. Griffin and J.D. Hauenstein. Real solutions to systems of polynomial equations and parameter continuation. *Adv. Geom.*, 15(2), 173–187, 2015.

[2] J.D. Hauenstein and F. Sottile. Algorithm 921: alphaCertified: certifying solutions to polynomial systems. *ACM Trans. Math. Software*, 38(4), 28, 2012.