Real solutions to Alt-Burmester problems

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The family of Alt-Burmester problems [3] consist of computing four-bar linkages which attain M poses and pass through N points, e.g., Figure 1 is a solution for $(M, N) = (0, 9)$ which is the classical Alt problem [1]. The classical Burmester problem [4] corresponds with $(M, N) = (5, 0)$.

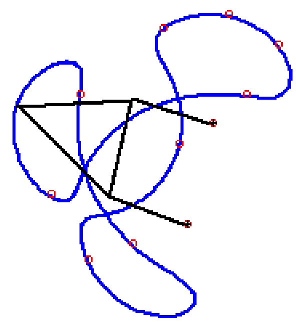

FIGURE 1. Four-bar linkage whose coupler curve passes through 9 given points

Since the initial pose can be trivially set, the classical Alt problem with $(M, N) = (0, 9)$ is the same as $(M, N) = (1, 8)$. The following table from [3] lists the Alt-Burmester problems which generically have finitely many solutions along to the corresponding polynomial system.

(M, N)	generic # of solutions
$(5, 0)$	$16 = 4 \cdot 4$
$(4, 2)$	60
$(3, 4)$	402
$(2, 6)$	2224
$(1, 8) = (0, 9)$	$8652 = 1442 \cdot 6$

This table highlights that the solutions to both the classical Burmester problem $(M, N) = (5, 0)$ and Alt problem $(M, N) = (1, 8)$ are actually in natural families of size 4 and 6, respectively.

Problem 1. *What is the maximum number of real solutions to each generically zero-dimensional Alt-Burmester problem?*

The classical Burmester problem can have all solutions real, while the other cases are open.

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