

Real Witness Points

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For an irreducible variety $V \subset \mathbb{C}^N$ of dimension r and degree d , a set $W \subset V$ is a *witness point set* for V if there exists a codimension r linear space $L \subset \mathbb{C}^N$ such that $W = V \cap L$ and $\#W = d$.

Problem 1. *Given V , what is the maximum number of real points in a witness point set?*

Two families of irreducible varieties are of particular interest: the Zariski closure of the special Euclidean group \mathcal{SE}_N [3] and the special orthogonal group \mathcal{SO}_N [1]. As described in [3], we can interpret the result of [2] which showed that all 40 assembly configurations of a Stewart-Gough platform can be real as showing \mathcal{SE}_3 has a witness point set consisting of $40 = \deg \mathcal{SE}_3$ real points.

Problem 2 (Problem 5.3 of [3]). *For $N \geq 4$, determine the maximum number of real witness points for \mathcal{SE}_N .*

The following is from [1].

Conjecture 1 (Conjecture 7.1 of [1]). *For any N , there exists a witness point set for \mathcal{SO}_N with all points real.*

REFERENCES

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- [3] J.D. Hauenstein, S.N. Sherman, and C.W. Wampler. Exceptional Stewart-Gough platforms, Segre embeddings, and the special Euclidean group. *SIAM J. Appl. Alg. Geom.*, 2(1), 179–205, 2018.