Homotopy Method for Bitangents and Tritangents

Jonathan D. Hauenstein

A general genus 3 curve is canonically represented as a quartic plane curve and has 28 bitangent lines, i.e., lines that are simultaneously tangent at two points on the curve. A general genus 4 curve is canonically represented as a space sextic (complete intersection in $\mathbb{P}^3$ of a quadric and cubic hypersurface) and has 120 tritangent planes, i.e., planes that are simultaneously tangent at three points on the curve.

Problem 1 (Modified from Problem 8 of [2]). *Develop a homotopy-based method for numerically computing bitangents and tritangents.*

A parameter homotopy [3] in the space of space quadrics and cubics was used in [1] to compute the 120 tritangents for various instances. However, this this required an "ab initio" solve to compute the solutions to the start system for this parameter homotopy. Hence, Problem 1 entails describing a homotopy-based method in which the start system has 28 and 120 easy to construct solutions, respectively, that can be deformed into the 28 and 120 bitangents and tritangents of a generic genus 3 and genus 4 curve, repsectively.

References

[1] J.D. Hauenstein, A. Kulkarni, E.C. Sertoz, and S.N. Sherman. Certifying reality of projections. *LNCS*, 10931, 200–208, 2018.

[2] A. Kulkarni, Y. Ren, M. Sayyary Namin and B. Sturmfels. Real space sextics and their tritangents. In *Proceedings of the 2018 ACM on International Symposium on Symbolic and Algebraic Computation*, ACM, New York, 2018, pp. 247–254.

[3] A.P. Morgan and A.J. Sommese. Coefficient-parameter polynomial continuation. *Appl. Math. Comput.* 29, 123–160, 1989.