ACMS 40790-01: Topics in Applied Mathematics
Nonlinear Equations, Spring 2016

Class website: http://www.nd.edu/~jhauenst/acms40790
2:00 – 2:50 pm MWF in Pasquerilla Center 105

Instructor: Jonathan Hauenstein (Hayes Healy 146, hauenstein@nd.edu)

Prerequisites: Applied Linear Algebra (ACMS 20620) or Linear Algebra (MATH 20610).

Rationale for Prerequisites: Linear algebra: Many techniques for solving nonlinear equations depend on solving linear equations, most notably is Newton's method. Additionally, many nonlinear equations are solved by "linearizing" the equations.

Recommended Books: *Introduction to Algebraic Geometry.* Hassett, 2007.
Numerically Solving Polynomial Systems with Bertini. Bates, Hauenstein, Sommese, and Wampler, 2013.

Additional Reading: *Ideals, Varieties, and Algorithms.* Cox, Little, and O'Shea, 2010.
Using Algebraic Geometry. Cox, Little, and O'Shea, 2005.
The Numerical Solution of Systems of Polynomials Arising in Engineering and Science. Sommese and Wampler, 2005.

Description: Solving linear and nonlinear systems of equations is a core part of applied and computational mathematics and there is a wide variety of solving techniques. When the equations are polynomial, computational methods in algebraic geometry can be used to describe their solution set. This course will introduce students to both algebraic methods (based on Grobner basis computations) as well as geometric methods (based on numerical algebraic geometry methods and homotopy continuation).

Topics: As time permits, this course will cover the following with examples from applications:

1. Review of solving linear equations and Newton's method.
2. Description of solution sets of nonlinear equations from both an algebraic and geometric viewpoint. This includes isolated solutions and positive-dimensional solution sets.
3. Multiplicity of solutions and ill-conditioning near singularities.
4. Computing Grobner bases of ideals.
5. Constructing homotopies and path tracking.
6. Computing a numerical irreducible decomposition.
7. Computing real solutions to systems of nonlinear equations.
8. Multilinear algebra and tensor decompositions.

Software: This course will use software to perform computations related to nonlinear equations. In class demonstrations will mainly use the free software packages Macaulay2 and Bertini along with MATLAB (available for free to students from oit.nd.edu).