

## Exam 1

September 2, 2020

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|----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
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MC. \_\_\_\_\_

9. \_\_\_\_\_

10. \_\_\_\_\_

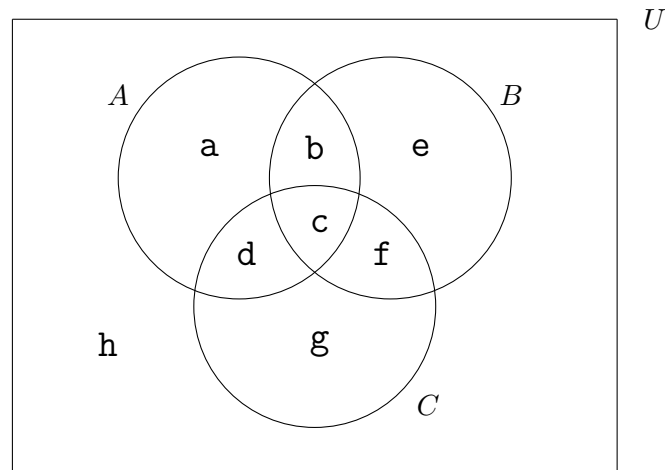
11. \_\_\_\_\_

12. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) In the following Venn diagram, which of the following is equal to  $A' \cup (B \cap C')$ ? (Note the “prime” over the  $A$  and the  $C$ .)



- (a)  $\{a, b, e, f, g, h\}$       (b)  $\{e\}$       (c)  $\{a, b, e, h\}$   
 (d)  $\{b, e, g, h\}$       (e)  $\{b, e, f, g, h\}$

2. (5 pts.) Over the course of the last month, I ate 50 pizzas. Of these, 35 had veggies on them, 18 had meat on them and 7 had no toppings. How many of them had both meat and veggies on them?

- (a) 10      (b) 7      (c) 33      (d) 8      (e) 25

3. (5 pts.) Claire has 5 mystery novels and Emily has 7 mystery novels (all different). They decide to go on a vacation together, and agree to bring two mystery novels each. In how many ways can they choose which 4 books they will take? (Note that the only issue is **which** two books each chooses, not what order they choose them.)

(a)  $C(12, 2)$

(b)  $C(5, 2) \cdot C(7, 2)$

(c)  $P(5, 2) \cdot P(7, 2)$

(d)  $C(5, 2) + C(7, 2)$

(e)  $P(5, 2) + P(7, 2)$

4. (5 pts.) A standard deck consists of 52 cards, with 13 cards in each of four suits (clubs, diamonds, hearts and spades). So there are four A's, four 2's, four 3's, etc. A "hand" is a subset of five cards. How many hands are **not** all of the same suit?

(a)  $C(52, 5) - C(13, 5)$

(b)  $C(52, 5) - 4 \cdot C(13, 5)$

(c)  $C(13, 1)^5$

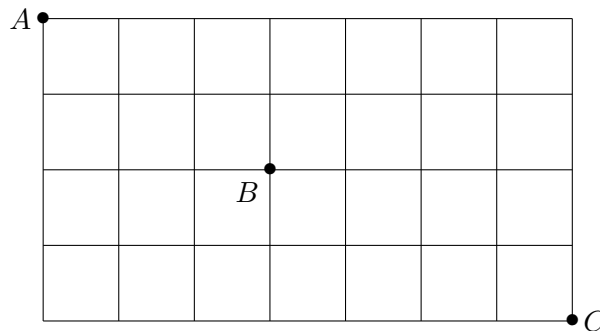
(d)  $4 \cdot P(13, 5)$

(e)  $C(52, 5) - 4 \cdot P(13, 5)$

5. (5 pts.) Mr. Chips is making up a true-false quiz for his class. He wants to put 8 questions in the quiz. In how many ways can he arrange it so that four of the answers are True and four are False?

- (a) 256                      (b) 1680                      (c) 4900                      (d) 70                      (e) 16

6. (5 pts.) The following grid is part of a street map of a city. Javier starts at point  $A$  and wants to get to point  $C$  going only to the right and down (on the map). However, along the way he wants to visit his brother Rafi, who is at point  $B$ . How many routes from  $A$  to  $C$  pass by  $B$ ?



- (a)  $C(11, 4) - C(5, 2)$                       (b)  $C(5, 2) + C(6, 2)$                       (c)  $C(5, 2) \cdot C(6, 2)$   
 (d)  $C(11, 4)$                       (e)  $2^{11}$

7. (5 pts.) The Aviation Club has 10 members. An anonymous donor offers them a free trip on a hot air balloon, but there are two conditions: At least one member of the club has to go (since someone has to control the balloon), and at most 8 can go (because of the capacity of the balloon). In how many ways can they decide who goes on the trip?

- (a) 1012            (b) 1024            (c) 1013            (d) 978            (e) 46

8. (5 pts.) Mary is planning to give Dave 28 DVD's for Christmas. Since that's a bit bulky, she decides to divide them into four groups of 7 to giftwrap. (Note that the order of these four groups is irrelevant.) In how many ways can she choose to divide them up?

- (a)  $\frac{28!}{7! \cdot 7! \cdot 7! \cdot 7!}$             (b)  $\frac{1}{24} \cdot \frac{28!}{7! \cdot 21!}$             (c)  $\frac{1}{24} \cdot \frac{28!}{7! \cdot 7! \cdot 7! \cdot 7!}$   
(d)  $\frac{28!}{7! \cdot 21!}$             (e)  $\frac{1}{4} \cdot \frac{28!}{7! \cdot 7! \cdot 7! \cdot 7!}$

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit!

Make sure that your answer is in the answer box. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

9. (15 pts.) A PIN number for a bank account consists of 6 digits (e.g. 629010). Your answers in this problem do not have to be numbers. You can use  $P(n, r)$ ,  $C(n, r)$ , exponents or factorials.

(a) How many PIN numbers are there if you are allowed to repeat digits?

**Answer to (a):**

(b) How many PIN numbers are there if you are not allowed to repeat digits?

**Answer to (b):**

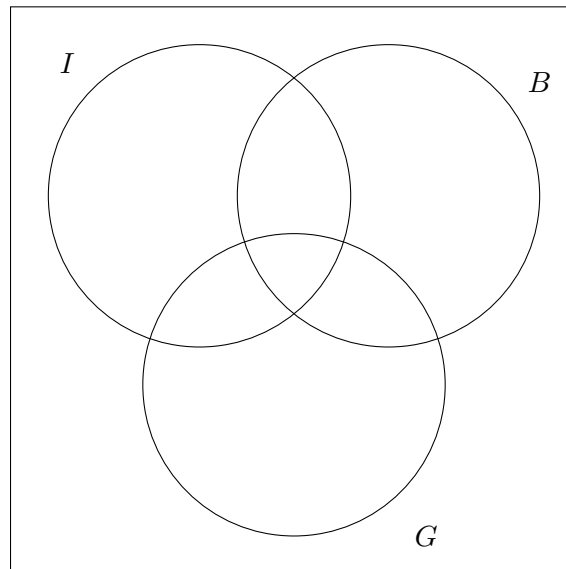
(c) How many PIN numbers have at least one repeated digit? [Hint: Think about what you did in the first two parts.]

**Answer to (c):**

10. (15 pts.) A group of 180 people decided to check **ancestry.com** for the preceding five generations to see what nationalities they found (not necessarily just one). Here is the relevant data:

- 40 have **ONLY** Italian ancestry (I).
- 15 have **ONLY** British ancestry (B).
- 25 have **ONLY** German ancestry (G).
- 20 have Italian and British ancestry but **NOT** German.
- 5 have British and German ancestry but **NOT** Italian.
- 30 have Italian and German ancestry but **NOT** British.
- 35 do **NOT** have any of the three ancestries.

Fill in **all** regions of the following Venn diagram.



11. (15 pts.) In this problem you will be looking at the word

*REGISTRATION.*

There are two R's, one E, one G, two I's, one S, two T's one A, one O and one N, for a total of 12 letters. For each part of the problem, be sure to explain your work and give a numerical answer.

(a) How many **different** 12-letter “words” can be made from these letters?

**Answer to (a):**

Remember this has to be a number!

(b) How many different 5-letter “words” can be made from these letters if we insist that each word consist of different letters? [Hint: how many different letters are there?]

**Answer to (b):**

Remember this has to be a number!



**12.** (15 pts.) A bag contains 14 colored marbles, of which 5 are red, 4 are white, 3 are blue and 2 are orange. (Assume that marbles of the same color are distinguishable from each other. In this problem, the order that you pick the marbles does not matter.) I plan to pick 3 marbles from the bag.

**Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).

(a) In how many total ways can I choose the three marbles if I don't care about the colors?

**Answer to (a):**

(b) In how many ways can I pick the three marbles so that they are all different colors?

**Answer to (b):**

(c) In how many ways can I pick the three marbles so that they are all the same color?

**Answer to (c):**

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