Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Fall 2020

Name:______________________________

Instructor: Juan Migliore

Exam 1

September 2, 2020

This exam is in two parts on 8 pages and contains 12 problems worth a total of 100 points. You have 1 hour and 30 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)

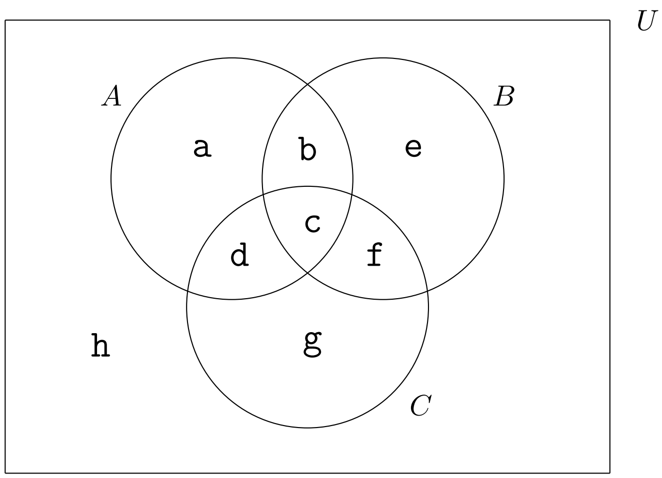
These are the answers to problems 1–8.

MC.	________
9.	________
10.	________
11.	________
12.	________
Tot.	________

Math 10120 Fall 2020, Exam 1 Initials:________

Multiple Choice

1. (5 pts.) In the following Venn diagram, which of the following is equal to $A' \cup (B \cap C')$? (Note the "prime" over the A and the C.)



(a) $\{a, b, e, f, g, h\}$ (b) $\{e\}$ (c) $\{a, b, e, h\}$

(d) $\{b, e, g, h\}$ (e) $\{b, e, f, g, h\}$

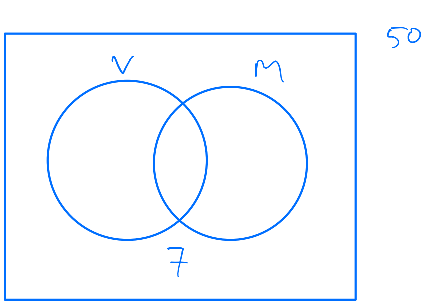
$B \cap C'$ means elements in B and not in C. That's $\{b, e\}$.

Then we take the union with A', ie throw in any element not in A.

All together $\{b, e, f, g, h\}$

2. (5 pts.) Over the course of the last month, I ate 50 pizzas. Of these, 35 had veggies on them, 18 had meat on them and 7 had no toppings. How many of them had both meat and veggies on them?

(a) 10 (b) 7 (c) 33 (d) 8 (e) 25



$50 - 7 = 43 = n(V \cup M)$

$43 = n(V \cup M) = n(V) + n(M) - n(V \cap M)$

$= 35 + 18 - n(V \cap M)$

So $n(V \cap M) = 35 + 18 - 43 = 10$

3. (5 pts.) Claire has 5 mystery novels and Emily has 7 mystery novels (all different). They decide to go on a vacation together, and agree to bring two mystery novels each. In how many ways can they choose which 4 books they will take? (Note that the only issue is **which** two books each chooses, not what order they choose them.)

(a) $C(12,2)$ (b) $C(5,2) \cdot C(7,2)$ (c) $P(5,2) \cdot P(7,2)$

(d) $C(5,2) + C(7,2)$ (e) $P(5,2) + P(7,2)$

Claire has $c(5,2)$ choices. Emily has $C(7,2)$ choices.

For each choice Claire could make, Emily has the same $c(7,2)$ choices.

So $c(5,2) \cdot c(7,2)$

4. (5 pts.) A standard deck consists of 52 cards, with 13 cards in each of four suits (clubs, diamonds, hearts and spades). So there are four A's, four 2's, four 3's, etc. A "hand" is a subset of five cards. How many hands are **not** all of the same suit?

(a) $C(52,5) - C(13,5)$ (b) $C(52,5) - 4 \cdot C(13,5)$

(c) $C(13,1)^5$ (d) $4 \cdot P(13,5)$

(e) $C(52,5) - 4 \cdot P(13,5)$

There are $c(52,5)$ possible hands, of which we exclude the ones consisting of cards all in the same suit.

For each suit there are $c(13,5)$ ways to choose 5 cards, and there are 4 suits. So

$c(52,5) - 4 \cdot c(13,5)$

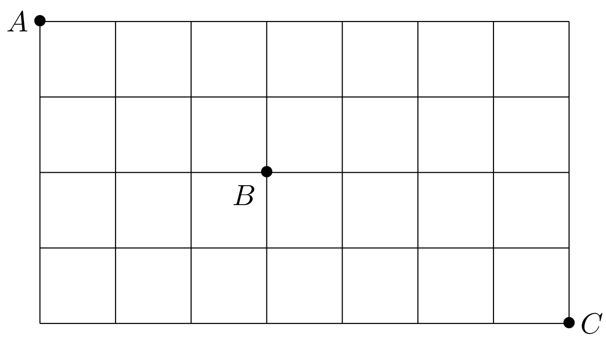
5. (5 pts.) Mr. Chips is making up a true-false quiz for his class. He wants to put 8 questions in the quiz. In how many ways can he arrange it so that four of the answers are True and four are False?

(a) 256 (b) 1680 (c) 4900 (d) 70 (e) 16

He just has to choose which 4 of the 8 questions are true (or equivalently which 4 are false).

$C(8,4) = \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2} = 70$

6. (5 pts.) The following grid is part of a street map of a city. Javier starts at point A and wants to get to point C going only to the right and down (on the map). However, along the way he wants to visit his brother Rafi, who is at point B. How many routes from A to C pass by B?



(a) $C(11,4) - C(5,2)$ (b) $C(5,2) + C(6,2)$ (c) $C(5,2)\cdot C(6,2)$

(d) $C(11,4)$ (e) 2^{11}

$C(5,2)$ routes from A to B

$C(6,2)$ routes from B to C.

$C(5,2)\cdot C(6,2)$ from A to C through B

7. (5 pts.) The Aviation Club has 10 members. An anonymous donor offers them a free trip on a hot air balloon, but there are two conditions: At least one member of the club has to go (since someone has to control the balloon), and and most 8 can go (because of the capacity of the balloon). In how many way can they decide who goes on the trip?

(a) 1012 (b) 1024 (c) 1013 (d) 978 (e) 46

There are 2^{10} subsets of a set with 10 elements. Of these we exclude

- sets with 0 elements (1)
- sets with 9 elements $(C(10,9)=10)$
- sets with 10 elements (1)

$1024 - 1 - 10 - 1 = 1012$

8. (5 pts.) Mary is planning to give Dave 28 DVD's for Christmas. Since that's a bit bulky, she decides to divide them into four groups of 7 to giftwrap. (Note that the order of these four groups is irrelevant.) In how many ways can she choose to divide them up?

(a) $\dfrac{28!}{7! \cdot 7! \cdot 7! \cdot 7!}$ (b) $\dfrac{1}{24} \cdot \dfrac{28!}{7! \cdot 21!}$ (c) $\dfrac{1}{24} \cdot \dfrac{28!}{7! \cdot 7! \cdot 7! \cdot 7!}$

(d) $\dfrac{28!}{7! \cdot 21!}$ (e) $\dfrac{1}{4} \cdot \dfrac{28!}{7! \cdot 7! \cdot 7! \cdot 7!}$

so it's an unordered partition

$\dfrac{1}{4!} \dfrac{28!}{7!7!7!7!}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit!
Make sure that your answer is in the answer box. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

9. (15 pts.) A PIN number for a bank account consists of 6 digits (e.g. 629010). Your answers in this problem do not have to be numbers. You can use $P(n, r)$, $C(n, r)$, exponents or factorials.

(a) How many PIN numbers are there if you are allowed to repeat digits?

$\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$ choices

Answer to (a):
10^6

(b) How many PIN numbers are there if you are not allowed to repeat digits?

$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}$

$= P(10, 6)$

Answer to (b):
$P(10, 6)$

(c) How many PIN numbers have at least one repeated digit? [Hint: Think about what you did in the first two parts.]

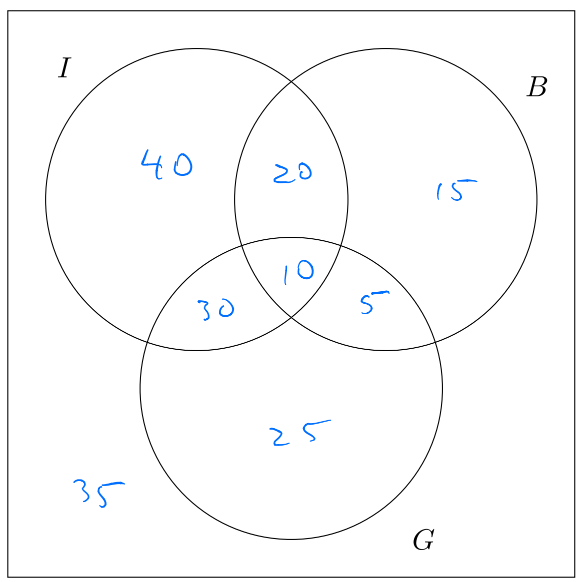
Exclude (b) from (a)

Answer to (c):
$10^6 - P(10, 6)$

10. (15 pts.) A group of 180 people decided to check `ancestry.com` for the preceding five generations to see what nationalities they found (not necessarily just one). Here is the relevant data:

- 40 have **ONLY** Italian ancestry (I).
- 15 have **ONLY** British ancestry (B).
- 25 have **ONLY** German ancestry (G).
- 20 have Italian and British ancestry but **NOT** German.
- 5 have British and German ancestry but **NOT** Italian.
- 30 have Italian and German ancestry but **NOT** British.
- 35 do **NOT** have any of the three ancestries.

Fill in **all** regions of the following Venn diagram.



180

You can enter the 40, 15 and 25 first.

Then the 20, 5, 30

Then the 35.

These add up to 170. So 180 − 170 = 10 are in the middle.

11. (15 pts.) In this problem you will be looking at the word

REGISTRATION.

There are two R's, one E, one G, two I's, one S, two T's one A, one O and one N, for a total of 12 letters. For each part of the problem, be sure to explain your work and give a numerical answer.

(a) How many **different** 12-letter "words" can be made from these letters?

There are 12 letters, so start with 12! But interchanging the two R's gives the same "word," and same with the I's and the T's.

So $\frac{12!}{2!\,2!\,2!}$

Answer to (a):

$\frac{12!}{2!\,2!\,2!}$

Remember this has to be a number!

(b) How many different 5-letter "words" can be made from these letters if we insist that each word consist of different letters? [Hint: how many different letters are there?]

There are 9 letters, and order is important, and we want 5 letters

Answer to (b):

$P(9,5)$

Remember this has to be a number!

12. (15 pts.) A bag contains 14 colored marbles, of which 5 are red, 4 are white, 3 are blue and 2 are orange. (Assume that marbles of the same color are distinguishable from each other. In this problem, the order that you pick the marbles does not matter.) I plan to pick 3 marbles from the bag.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) In how many total ways can I choose the three marbles if I don't care about the colors?

Answer to (a):

$C(14,3)$

(b) In how many ways can I pick the three marbles so that they are all different colors?

There are 4 colors and we only pick 3 so we have to look at all ways to exclude one.

$$\left.\begin{array}{ll} RWB & 5\cdot 4\cdot 3 \\ RWO & 5\cdot 4\cdot 2 \\ RBO & 5\cdot 3\cdot 2 \\ WBO & 4\cdot 3\cdot 2 \end{array}\right\} \text{add these up}$$

Answer to (b):

154

(c) In how many ways can I pick the three marbles so that they are all the same color?

All red: $C(5,3) = 10$

All white: $C(4,3) = 4$

All blue $C(3,3) = 1$

All orange: 0 (not enough to get 3)

Answer to (c):

15