Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Fall 2020

Name:________________________________

Instructor: Juan Migliore

Exam 2

September 30, 2020

This exam is in two parts on 8 pages and contains 12 problems worth a total of 100 points. You have 1 hour to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

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Place an $\times$ through your answer to each problem.

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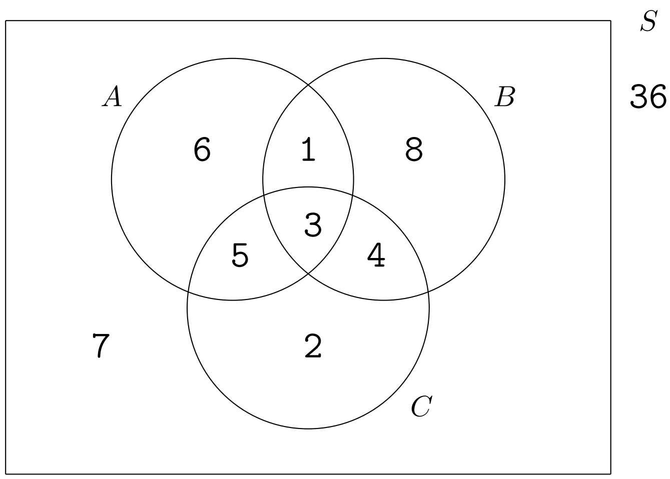
MC.	________
9.	________
10.	________
11.	________
12.	________
Tot.	________

Multiple Choice

1. (5 pts.) The 36 members of the Travel Club have upcoming trips planned to Argentina (A), Bolivia (B), and Columbia (C). The following Venn diagram indicates how many members of the club are planning to go on the corresponding trips. If a student is chosen at random, what is the probability the she is going to Argentina?



(a) $\frac{8}{36}$ (b) $\frac{16}{36}$ (c) $\frac{6}{36}$ (d) $\frac{15}{36}$ (e) $\frac{12}{36}$

All together we have 36 members. Of them, 6+1+3+5=15 are going to Argentina

2. (5 pts.) Refer to the club and Venn diagram from Problem #1. A student is chosen at random, and it is discovered that he is going to Bolivia. With this extra information, what is the probability that he is also going to Argentina?

(a) $\frac{1}{16}$ (b) $\frac{15}{36}$ (c) $\frac{4}{8}$ (d) $\frac{3}{16}$ (e) $\frac{4}{16}$

There are 16 going to Bolivia. Of them, 1+3=4 are also going to Argentina

3. (5 pts.) Suppose you roll a red die and a blue die, and record the sum of the numbers on the two dice. What is the probability that the sum is greater than or equal to seven?

(a) $\frac{1}{2}$ (b) $\frac{7}{12}$ (c) $\frac{5}{12}$ (d) $\frac{1}{6}$ (e) $\frac{1}{3}$

The 36 possible (red, blue) outcomes are

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

The red ones have a sum ≥ 7. There are 21 of them. $\frac{21}{36} = \frac{7}{12}$

4. (5 pts.) Suppose you flip a coin 5 times. What is the probability that you get at least two Heads?

(a) $\frac{27}{32}$ (b) $\frac{31}{32}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$ (e) $\frac{13}{16}$

There are $2^5 = 32$ possible sequences of H and T.

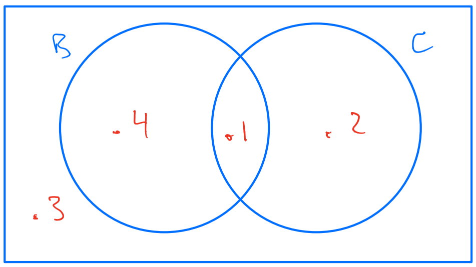
of them, $\binom{5}{1}$ ways to get just one H and 1 way to get no H.

So $1 - \frac{5+1}{32} = \frac{26}{32} = \frac{13}{16}$

5. (5 pts.) At a certain high school, 50% of the students are in the marching band, 30% of students are in the choir, and 30% are in neither. What is the probability that a randomly chosen person is in the band, but **NOT** in the choir?

(a) 0.5 (b) 0.2 (c) 0.6 (d) 0.4 (e) 0.3

B = band, C = choir



Since 30% are in neither, we have

$P(B) = .5$, $P(C) = .3$ $P(B \cup C) = 1 - .3 = .7$

Since $P(B \cup C) = P(B) + P(C) - P(B \cap C)$,

we get $P(B \cap C) = .5 + .3 - .7 = .1$

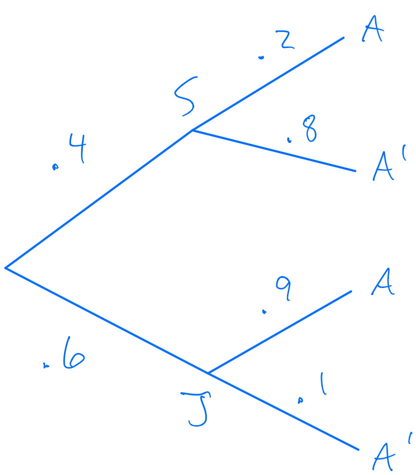
Then we fill in the regions as done in the Venn diagram. So

$P(B \cap C') = .4$

6. (5 pts.) A class at a high school is composed of 40% sophomores and 60% juniors. On an exam, 20% of the sophomores and 90% of the juniors got an A. A student is selected at random and found to have gotten an A. What is the probability that they are a sophomore?

(a) $\dfrac{(0.4)(0.2)}{(0.4)(0.2) + (0.6)(0.9)}$ (b) $(0.4)(0.2)$ (c) $\dfrac{(0.4)(0.2)}{(0.6)(0.9)}$

(d) $\dfrac{(0.4)(0.2)}{(0.4)(0.2) + (0.4)(0.8)}$ (e) 0.4



$$P(S|A) = \frac{P(S \cap A)}{P(A)}$$

$$= \frac{(.4)(.2)}{(.4)(.2) + (.6)(.9)}$$

7. (5 pts.) Which of the following is **NOT** a way to show that two events E and F are independent?

(a) Show that $P(E \cup F) = P(E) + P(F)$.

(b) Show that $P(E \mid F) = P(E)$.

(c) Show that $P(F \mid E) = P(F)$.

(d) Show that $P(E \cap F) = P(E) \cdot P(F)$.

(e) Show that E happening does not affect the probability of F happening.

8. (5 pts.) You are dealt 3 cards sequentially from a standard deck of 52 cards **without replacement**. What is the probability that the first two cards are aces and the third card is a king?

(a) $\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50}$

(b) $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50}$

(c) $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{4}{50}$

(d) $C(4,2) \cdot 4$

(e) $\left(\frac{4}{52}\right)^3$

1st card ace : $\frac{4}{52}$

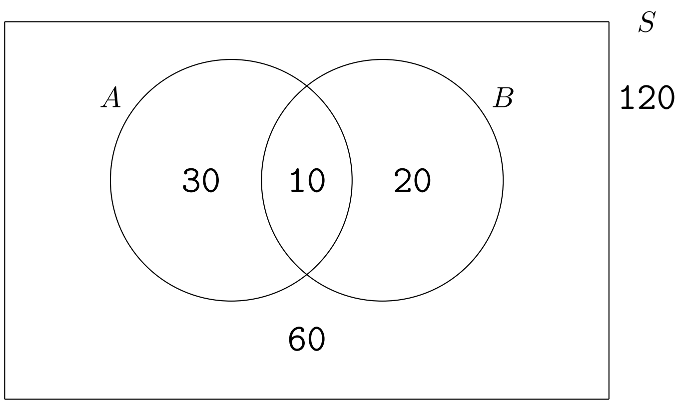
2nd card ace : $\frac{3}{51}$

3rd card king $\frac{4}{50}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit!
Make sure that your answer is in the answer box. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

9. (15 pts.) The events A and B contained in the sample space S are modeled by the following Venn diagram.



(a) Are A and B independent events? You must show all of your work for full credit.

We have to show $P(A|B) = P(A)$ or $P(A \cap B) = P(A)P(B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{10/120}{30/120} = \frac{1}{3}$

$P(A) = \frac{40}{120} = \frac{1}{3}$ ✓

So yes, they are independent

$P(A \cap B) = \frac{10}{120}$

$P(A) = \frac{40}{120}$ $P(B) = \frac{30}{120}$

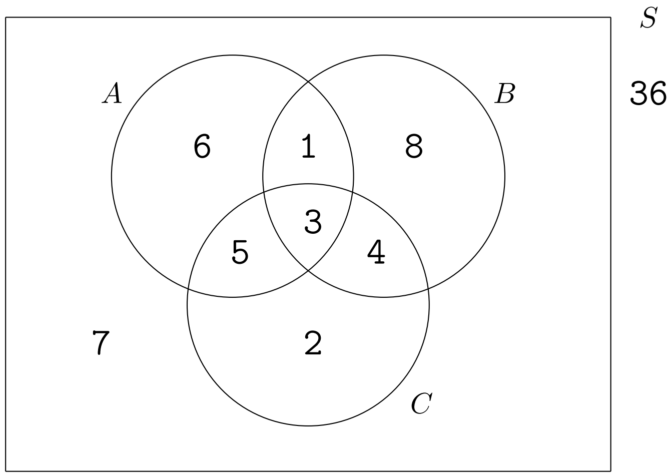
$\frac{10}{120} = \frac{40}{120} \cdot \frac{30}{120}$

$\frac{1}{12} = \frac{1}{3} \cdot \frac{1}{4}$ ✓

(b) Are A and B mutually exclusive? Explain why or why not.

Mutually exclusive means the sets are disjoint, i.e. they have no elements in common. Since here there are 10 elements in common, they are not mutually exclusive.

10. (15 pts.) The following Venn diagram describes the relative sizes of events A, B, and C contained in a sample space S.



Find each of the probabilities using the numbers in the diagram. For example, if we asked for $P(A)$, please write

$$\frac{6+1+3+5}{36}.$$

You do not have to simplify this to $\frac{15}{36} = \frac{5}{12}$ unless you want to.

(a) $P(A \cup B) =$

$$\frac{6+1+3+5+8+4}{36} = \frac{27}{36} = \frac{3}{4}$$

(b) $P(A \cap B' \cap C) =$

(in A and in C but not in B)

$$\frac{5}{36}$$

(c) $P(A \mid B) =$ $\frac{1+3}{1+8+3+4}$

$$= \frac{4}{16} = \frac{1}{4}$$

(d) $P(A \cup C \mid B) =$ $\frac{1+3+4}{1+8+3+4} = \frac{1}{2}$

(e) $P(B \mid A \cup C) =$ $\frac{1+3+4}{6+1+3+5+4+2}$

$$= \frac{8}{21}$$

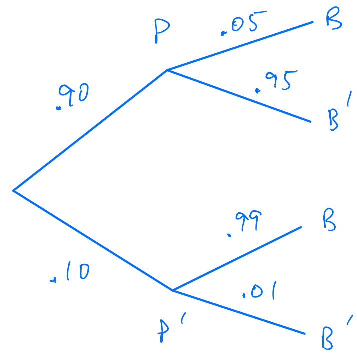
11. (15 pts.) In the city of Boston, 90% of people are Patriots fans, and 10% are not.

- Of the Patriots fans, 5% believe the Patriots cheated by deflating the footballs, and 95% do not.
- Of the non-Patriots fans, 99% believe the Patriots cheated by deflating the footballs, and 1% do not.

(a) Draw a tree diagram representing this situation. Be sure to explain your notation and include all the probabilities.

P = patriot fan

B = believe



(b) If a person in Boston is chosen at random, what is the probability that they do **not** believe the Patriots cheated?

$$P(B') = (.90)(.95) + (.10)(.01)$$

$$= .855 + .001$$

$$= .856$$

4
.95
.9
.855

(c) A person in Boston is chosen at random, and it is determined that they believe the Patriots cheated. What is the probability that they are a Patriots fan?

$$P(P \mid B) = \frac{P(P \cap B)}{P(B)}$$

$$= \frac{(.90)(.05)}{(.90)(.05) + (.10)(.99)}$$

$$= \frac{.045}{.045 + .099}$$

$$= .3125$$

12. (15 pts.) A bag contains 12 colored marbles: 5 red marbles, 4 blue marbles, and 3 white marbles. We randomly select 3 marbles from the bag **without replacement**.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) Assuming that order in which the marbles are drawn is not important, what is the probability that all three marbles are different colors?

$$\frac{C(5,1)\cdot C(4,1)\cdot C(3,1)}{C(12,3)}$$

For each of the C(5,1) ways to choose the red ball, there are C(4,1) ways to choose the blue and then C(3,1) ways to choose the white one.

all ways of choosing 3 of the 12 balls

Answer to (a): $\dfrac{C(5,1)\cdot C(4,1)\cdot C(3,1)}{C(12,3)}$

(b) Assuming that order **is** important, what is the probability that the first marble drawn is red, the second marble is blue, and the third is white?

Answer to (b): $\dfrac{5}{12}\cdot\dfrac{4}{11}\cdot\dfrac{3}{10}$

1st ball 2nd ball 3rd ball

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