

### Exam 3

November 6, 2020

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**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.** The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an  $\times$  through your answer to each problem.

- |    |     |     |     |     |     |
|----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
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| 8. | (a) | (b) | (c) | (d) | (e) |

MC. \_\_\_\_\_

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12. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) A 10-point quiz was given to a class of 20 students, and the relative frequency of each score is given in the table below.

score	10	9	8	7	6
relative frequency	0.1	0.35	0.25	0.2	0.1

How many students scored 7 points on the quiz?

- (a) 2                      (b) 0.2                      (c) 1                      (d) 4                      (e) 7

2. (5 pts.) An exam is given to a class of 9 students, and the average score is 80 points (out of 100). The next day, a tenth student joins the class and is given the exam. This student gets a score of 69. What is the new average for the 10-student class? Round to the nearest full point.

- (a) 79                      (b) 83                      (c) 92                      (d) 80  
(e) There is not enough information given.

**3.** (5 pts.) A box contains 6 red balls and 6 blue balls. A ball is drawn from the box. If the ball is blue, you stop. If the ball is red, you throw it away (not back into the box) and draw TWO balls.

If one (or both) of these balls is blue, you stop. If both of these balls are red, you throw these two balls away and draw THREE balls.

This procedure is repeated (with the number of balls drawn increasing by one each time) until one or more **blue** balls are drawn. Let  $X$  be the random variable given by the total number of balls drawn before stopping. What are the possible values for  $X$ ?

- (a)  $\{1, 2, 3, 4, 5, 6, 7\}$                       (b)  $\{1, 3, 6, 10\}$                       (c)  $\{1, 2, 3, \dots, 11, 12\}$   
(d)  $\{7\}$                                       (e)  $\{1, 2, 3, 4\}$

**4.** (5 pts.) Suppose a random variable has the following probability distribution:

$x_i$	$p_i$
5	0.2
10	0.6
15	0.2

Calculate the standard deviation  $\sigma(X)$  for  $X$  (rounded to two decimal places).

- (a) 3.74                      (b) 0                      (c) 1.10                      (d) 7.28                      (e) 3.16

5. (5 pts.) LeBron James has a lifetime free throw average of 75%. Based only on this (not taking into account adrenalin, pressure, injuries, etc.), suppose that he shoots 6 free throws in some game. What's the probability that he successfully makes **AT LEAST** 5 of those shots?

- (a)  $C(6, 5)(.75)^5(.25)$                       (b)  $1 - C(6, 5)(.75)^5(.25) - (.75)^6$   
(c)  $C(6, 5)(.75)^5(.25) + (.75)^6$                       (d)  $1 - C(6, 5)(.75)^5(.25)$   
(e)  $A(6.5) - A(4.5)$

6. (5 pts.) In any normal distribution, what percentage of scores lie **AT MOST** two standard deviations from the mean? (Be sure to consider both **below** the mean and **above** the mean.) Use the table at the end of this exam.

- (a) 95.46%              (b) 47.73%              (c) 68.26%              (d) 34.13%              (e) 25.00%

7. (5 pts.) Everyone in the country of Freedonia lives on the shore of Lake Groucho and rows to work every day in a rowboat. The amount of time it takes them to row to work is normally distributed, with a mean of 75 minutes and a standard deviation of 10 minutes. What fraction of the population gets to work in under an hour (60 minutes)? Use the table at the end of this exam.

- (a) 43.32%      (b) 6.68%      (c) 56.68%      (d) 1.50%      (e) 93.32%

8. (5 pts.) Doug's diet consists exclusively of ravioli and lasagna, which he keeps in his freezer until he cooks a box. Since his freezer is completely empty, he has gone to the food store to buy the next week's supply of pasta. Each box of ravioli measures 20 cubic inches and costs \$6; each box of lasagna measures 30 cubic inches and costs \$7. On doctor's orders, each week he has to eat at least twice as many boxes of ravioli as lasagna. His freezer measures 10,000 cubic inches, and his weekly budget is \$800. If  $x$  is the number of boxes of ravioli he buys for the coming week and  $y$  is the number of boxes of lasagna he buys for the coming week, what system of linear inequalities is represented by this information?

- (a) 
$$\begin{array}{l} 20x + 6y \leq 10,000 \\ 30x + 7y \leq 800 \\ x - 2y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{array}$$
- (b) 
$$\begin{array}{l} 20x + 30y \leq 10,000 \\ 6x + 7y \leq 800 \\ 2x - y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{array}$$
- (c) 
$$\begin{array}{l} 20x + 6y \leq 10,000 \\ 30x + 7y \leq 800 \\ 2x - y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{array}$$
- (d) 
$$\begin{array}{l} 20x + 30y \leq 800 \\ 6x + 7y \leq 10,000 \\ 2x - y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{array}$$
- (e) 
$$\begin{array}{l} 20x + 30y \leq 10,000 \\ 6x + 7y \leq 800 \\ x - 2y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{array}$$

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit!

Make sure that your answer is in the answer box. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

9. (15 pts.) Suppose a box contains 6 balls: two labelled 1, two labelled 2, and two labelled 3. Two balls are selected at random from the box. Let  $X$  be the random variable given by the **larger** value of the two balls (if both balls have the value, then  $X$  is equal to this common value).

(a) What are the possible values for  $X$ ?

**Answer:**

(b) Find the probability distribution for  $X$ . (Numerical values, please.)

**Answer:**

$X$	$P(X)$

(c) **This part is completely unrelated to parts (a) and (b)!** Calculate the expected value of the random variable  $X$  with the following probability distribution:

$X$	$P(X)$
0	0.1
2	0.2
4	0.3
6	0.4

**Answer:**

**10.** (15 pts.) Ten students are sampled on campus, and asked how many classes they are enrolled in this semester. The results are:

3, 3, 3, 4, 4, 4, 4, 5, 5, 5.

(a) Calculate the average number of classes taken by these students.

**Answer:**

(b) Calculate the **sample** variance and standard deviation for this data set.

**Answer:**

$$s^2 =$$

$$s =$$

**11.** (15 pts.) Mercury Airlines reports that when someone buys a ticket, there is only an 80% probability that the person will show up for the flight. For the June 22 flight to Manchester, they sold 200 tickets.

- (a) Find the probability that **exactly** 170 of the ticket purchasers show up for the flight. For this problem you don't have to give a numerical answer – it's ok to leave it in terms of binomial coefficients and powers.

**Answer:**

For parts (b) and (c) we would like a numerical answer.

- (b) Find the mean and the standard deviation for this Bernoulli experiment.

**Answer:**

$$\mu =$$

$$\sigma =$$

- (c) Use the **normal approximation to the binomial distribution** to estimate the probability that **at least** 170 of the ticket purchasers show up for the flight.

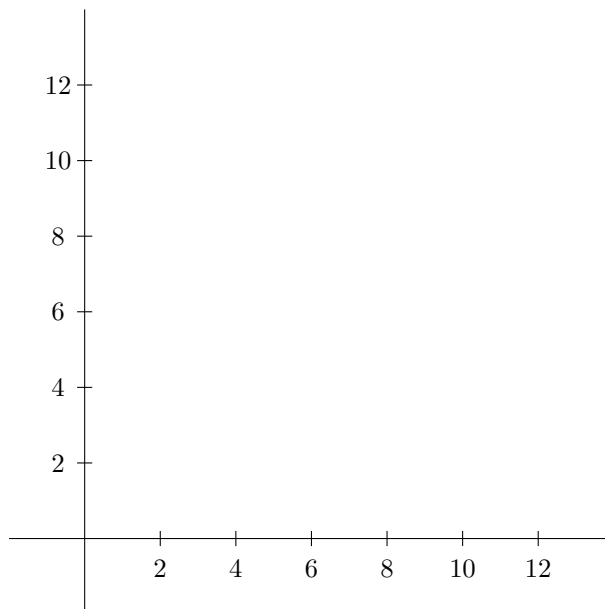
**Answer:**



12. (15 pts.) Consider the system of linear inequalities

$$\begin{aligned}x + 2y &\leq 10 \\2x + y &\leq 8 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

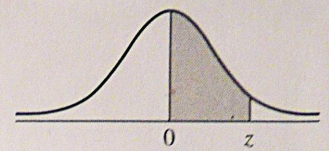
(a) Graph the feasible region using the axes provided. **Be sure to label the lines and the corners of the feasible region, and shade the feasible region.**



(b) Find the maximum value of the objective function  $3x - 4y$  on the feasible set described in (a). Be sure to show your work.



# Area Under the Standard Normal Curve



$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332	2.00	0.4773	2.50	0.4938	3.00	0.4987
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345	2.01	0.4778	2.51	0.4940	3.01	0.4987
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357	2.02	0.4783	2.52	0.4941	3.02	0.4987
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370	2.03	0.4788	2.53	0.4943	3.03	0.4988
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382	2.04	0.4793	2.54	0.4945	3.04	0.4988
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394	2.05	0.4798	2.55	0.4946	3.05	0.4989
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406	2.06	0.4803	2.56	0.4948	3.06	0.4989
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418	2.07	0.4808	2.57	0.4949	3.07	0.4989
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4430	2.08	0.4812	2.58	0.4951	3.08	0.4990
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441	2.09	0.4817	2.59	0.4952	3.09	0.4990
0.10	0.0398	0.60	0.2258	1.10	0.3643	1.60	0.4452	2.10	0.4821	2.60	0.4953	3.10	0.4990
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463	2.11	0.4826	2.61	0.4955	3.11	0.4991
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474	2.12	0.4830	2.62	0.4956	3.12	0.4991
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4485	2.13	0.4834	2.63	0.4957	3.13	0.4991
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495	2.14	0.4838	2.64	0.4959	3.14	0.4992
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505	2.15	0.4842	2.65	0.4960	3.15	0.4992
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515	2.16	0.4846	2.66	0.4961	3.16	0.4992
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525	2.17	0.4850	2.67	0.4962	3.17	0.4992
0.18	0.0714	0.68	0.2518	1.18	0.3810	1.68	0.4535	2.18	0.4854	2.68	0.4963	3.18	0.4993
0.19	0.0754	0.69	0.2549	1.19	0.3830	1.69	0.4545	2.19	0.4857	2.69	0.4964	3.19	0.4993
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554	2.20	0.4861	2.70	0.4965		
0.21	0.0832	0.71	0.2612	1.21	0.3869	1.71	0.4564	2.21	0.4865	2.71	0.4966		
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573	2.22	0.4868	2.72	0.4967		
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582	2.23	0.4871	2.73	0.4968		
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591	2.24	0.4875	2.74	0.4969		
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599	2.25	0.4878	2.75	0.4970		
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608	2.26	0.4881	2.76	0.4971		
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616	2.27	0.4884	2.77	0.4972		
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625	2.28	0.4887	2.78	0.4973		
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633	2.29	0.4890	2.79	0.4974		
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641	2.30	0.4893	2.80	0.4974		
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649	2.31	0.4896	2.81	0.4975		
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656	2.32	0.4898	2.82	0.4976		
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664	2.33	0.4901	2.83	0.4977		
0.34	0.1331	0.84	0.2996	1.34	0.4099	1.84	0.4671	2.34	0.4904	2.84	0.4977		
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678	2.35	0.4906	2.85	0.4978		
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686	2.36	0.4909	2.86	0.4979		
0.37	0.1443	0.87	0.3079	1.37	0.4147	1.87	0.4693	2.37	0.4911	2.87	0.4980		
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4700	2.38	0.4913	2.88	0.4980		
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706	2.39	0.4916	2.89	0.4981		
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713	2.40	0.4918	2.90	0.4981		
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719	2.41	0.4920	2.91	0.4982		
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726	2.42	0.4922	2.92	0.4983		
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732	2.43	0.4925	2.93	0.4983		
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738	2.44	0.4927	2.94	0.4984		
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744	2.45	0.4929	2.95	0.4984		
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750	2.46	0.4931	2.96	0.4985		
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756	2.47	0.4932	2.97	0.4985		
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4762	2.48	0.4934	2.98	0.4986		
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767	2.49	0.4936	2.99	0.4986		



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|----|------------------|------------------|------------------|------------------|------------------|
| 1. | (a)              | (b)              | (c)              | ( <del>d</del> ) | (e)              |
| 2. | ( <del>a</del> ) | (b)              | (c)              | (d)              | (e)              |
| 3. | (a)              | ( <del>b</del> ) | (c)              | (d)              | (e)              |
| 4. | (a)              | (b)              | (c)              | (d)              | ( <del>e</del> ) |
| 5. | (a)              | (b)              | ( <del>c</del> ) | (d)              | (e)              |
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| 7. | (a)              | ( <del>b</del> ) | (c)              | (d)              | (e)              |
| 8. | (a)              | (b)              | (c)              | (d)              | ( <del>e</del> ) |

MC. \_\_\_\_\_

9. \_\_\_\_\_

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Tot. \_\_\_\_\_