

Exam 3

April 18, 2019

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. _____

11. _____

12. _____

13. _____

14. _____

15. _____

Tot. _____

Multiple Choice

1. (5 pts.) The 50 students in Math 456 took a 10-point quiz, and the following gives the number of students scoring 10, 9, 8, 7 and 6. (No other scores were obtained.)

score	10	9	8	7	6
number of students	22	12	6	7	3

Find the **relative frequency** of the score "10."

- (a) 0.22 (b) 0.44 (c) 22 (d) 0.2 (e) 0.1

$$\frac{22}{50} = \frac{44}{100} = 0.44$$

2. (5 pts.) There are six students shooting baskets. The first five made 10, 8, 11, 6 and 13 baskets, respectively. How many baskets must the sixth student make in order for the average (mean) number for all six to be 10?

- (a) 10 (b) 11 (c) 12 (d) 13 (e) 14

$$\frac{10 + 8 + 11 + 6 + 13 + x}{6} = 10$$

$$48 + x = 60$$

$$x = 12$$

3. (5 pts.) Find the population variance for the following set of data (up to 2 decimal places). [Note that this is asking for the variance, **not** the standard deviation.]

11, 13, 8, 14, 9.

- (a) 4.8 (b) 0 (c) 2.28 (d) 26 (e) 5.2

x	$x - \mu$	$(x - \mu)^2$
11	0	0
13	2	4
8	-3	9
14	3	9
9	-2	4
55		26

$$\mu = \frac{55}{5} = 11 \qquad \sigma^2 = \frac{26}{5} = 5.2$$

4. (5 pts.) A box contains 8 blue balls, 6 red balls and 4 green balls, all mixed together. A ball is selected randomly from the urn. If it is green, you stop. If it is not green, you throw it away (NOT back in the urn) and select another one randomly. This is repeated until you select a green ball. Let X denote the random variable that gives the number of times you selected a ball. (For example, if you didn't get a green ball on the first selection but did on the second selection then $X = 2$ because you drew a total of two balls.) What values may X assume?

- (a) {1, 2, 3, ..., 15} (b) {2, 3, ..., 15} (c) {1, 2, 3, ..., 14}
- (d) {1, 2, 3, ..., 18} (e) {2, 3, ..., 14}

The fewest you would need to get a green is 1.
 The most you would need is $8 + 6 + 1 = 15$
 you could have anything in between.

5. (5 pts.) An urn contains three balls marked “1,” three balls marked “2” and three balls marked “3.” Claire selects two balls at random. Let X be the sum of the numbers on the two selected balls. Find the probability distribution for X . [Hint: notice that $4 = 1 + 3 = 2 + 2$.]

(a)

x	$P(x)$
2	2/36
3	8/36
4	16/36
5	8/36
6	2/36

(b)

x	$P(x)$
2	4/36
3	9/36
4	10/36
5	9/36
6	4/36

(c)

x	$P(x)$
2	3/36
3	8/36
4	14/36
5	8/36
6	3/36

(d)

x	$P(x)$
2	4/36
3	8/36
4	12/36
5	8/36
6	4/36

(e)

x	$P(x)$
2	3/36
3	9/36
4	12/36
5	9/36
6	3/36

The sum is at least 2 and at most 6.

Note $C(9, 2) = 36$

	x	$P(x)$
1+1	2	$C(3, 2)/36 = 3/36$
1+2	3	$C(3, 1) \cdot C(3, 1)/36 = 9/36$
1+3 or 2+2	4	$[C(3, 1) \cdot C(3, 1) + C(3, 2)]/36 = 12/36$
2+3	5	$C(3, 1) \cdot C(3, 1)/36 = 9/36$
3+3	6	$C(3, 2)/36 = 3/36$

6. (5 pts.) Compute the **standard deviation** $\sigma(X)$ for the random variable defined as follows (to two decimal places):

x_i	p_i
10	0.3
20	0.2
30	0.5

- (a) 8.72 (b) 22 (c) 0 (d) 8.84 (e) 8.93

$$\mu = 10(0.3) + 20(0.2) + 30(0.5) = 3 + 4 + 15 = 22$$

x_i	p_i	$x_i - \mu$	$(x_i - \mu)^2$	$p_i(x_i - \mu)^2$
10	0.3	-12	144	43.2
20	0.2	-2	4	0.8
30	0.5	8	64	32.0

76.0

$$\sigma = \sqrt{76} = 8.72$$

7. (5 pts.) Some data has a mean of $\mu = 32$ and standard deviation $\sigma = 2.5$. Find the z -score for a score of 28 (up to 2 decimal places).

- (a) 1.6 (b) -4 (c) -2 (d) -1.6 (e) 2

$$\frac{28 - 32}{2.5} = \frac{-4}{2.5} = -1.6$$

8. (5 pts.) A multiple choice exam has 10 questions, each containing 5 possible answers. Lee randomly guesses all the answers. You fail if you get 0 to 7 right and pass if you get 8 to 10 right. What is the probability that Lee will pass the exam?

- (a) $\left(\frac{1}{5}\right)^8 \left(\frac{4}{5}\right)^2 + \left(\frac{1}{5}\right)^9 \left(\frac{4}{5}\right)^1 + \left(\frac{1}{5}\right)^{10} \left(\frac{4}{5}\right)^0$
- (b) $C(10, 8) \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + C(10, 9) \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + C(10, 10) \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$
- (c) $C(10, 8) \left(\frac{1}{5}\right)^8 \left(\frac{4}{5}\right)^2 + C(10, 9) \left(\frac{1}{5}\right)^9 \left(\frac{4}{5}\right)^1 + C(10, 10) \left(\frac{1}{5}\right)^{10} \left(\frac{4}{5}\right)^0$
- (d) $\left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$
- (e) $C(10, 8) \left(\frac{4}{5}\right)^8 \left(\frac{1}{5}\right)^2 + C(10, 9) \left(\frac{4}{5}\right)^9 \left(\frac{1}{5}\right)^1 + C(10, 10) \left(\frac{4}{5}\right)^{10} \left(\frac{1}{5}\right)^0$

$$p = \frac{1}{5}$$

$$q = \frac{4}{5}$$

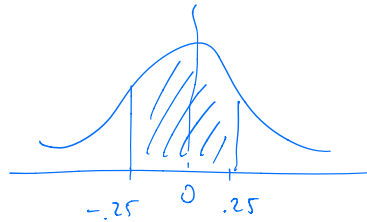
$$n = 10$$

9. (5 pts.) The weights of the members of some population are normally distributed with a mean of 30 lbs and a standard deviation of 8 lbs. Let X be the random variable corresponding to this normal distribution. If a member of that population is chosen at random, find $P(28 \leq X \leq 32)$.

- (a) 9.87% (b) 98.76% (c) 49.38% (d) 19.74% (e) 38.30%

$$z_1 = \frac{28 - 30}{8} = \frac{-2}{8} = -0.25$$

$$z_2 = \frac{32 - 30}{8} = \frac{2}{8} = 0.25$$



$$\begin{aligned} &A(0.25) + A(0.25) \\ &= 2(0.0987) \\ &= 0.1974 = 19.74\% \end{aligned}$$

10. (5 pts.) Which of the following points is in the feasible region for the following system of inequalities? (Be careful with \leq versus $<$ and with \geq versus $>$.)

$$\begin{aligned} 2x + 3y &\geq 4 \\ 5x - 6y &< 4 \\ x \geq 0, \quad y &\geq 0 \end{aligned}$$

- (a) (2,1) (b) (1,1) (c) (-1,2) (d) (0,1) (e) (2,0)

✓
fails $5x - 6y < 4$

↓
yes
 $5 \geq 4$
 $-1 < 4$
 $x \geq 0, y \geq 0$

✓
fails $x \geq 0$

↓
fails $2x + 3y \geq 4$

↓
fails $5x - 6y < 4$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) A shelf contains 10 CDs, of which 6 are Beatles and 4 are Simon and Garfunkel. These CDs are all mixed together, and Emily randomly chooses two to take with her on a trip. Let X be the random variable counting **the number of Beatles CDs that she picks**.

(a) What are the possible values that X may take?

0, 1, 2

(b) Find the probability distribution for X .

Note $c(10, 2) = 45$

X	P
0	$c(4, 2) / c(10, 2) = 6/45$
1	$c(6, 1) \cdot c(4, 1) / c(10, 2) = 24/45$
2	$c(6, 2) / c(10, 2) = 15/45$

(c) What is the expected value for this probability distribution? [It is not necessarily going to be an integer.]

$$0 \cdot \frac{6}{45} + 1 \cdot \frac{24}{45} + 2 \cdot \frac{15}{45} = \frac{24 + 30}{45} = \frac{54}{45} = \frac{6}{5} = 1.2$$

12. (10 pts.) Exactly 10% of a certain population own a Harry Potter book. Suppose that 10,000 people are chosen at random and asked if they own a Harry Potter book.

[Note: the formulas

$$\begin{aligned}\mu &= np \\ \sigma &= \sqrt{npq}\end{aligned}$$

may be useful in this problem.]

- (a) (3 points) Find the mean and standard deviation of this binomial distribution.

$$n = 10,000$$

$$p = 0.1$$

$$q = 0.9$$

$$\mu = (10,000)(0.1) = 1,000$$

$$\sigma = \sqrt{(10,000)(0.1)(0.9)} = \sqrt{900} = 30$$

- (b) (7 points) Use the normal approximation to the binomial distribution to estimate the probability that between 909 and 1037 of the 10,000 people that were randomly chosen own a Harry Potter book.

$$P(908.5 \leq X \leq 1037.5)$$

$$x_1 = 908.5 \rightsquigarrow z_1 = \frac{908.5 - 1000}{30} = \frac{-91.5}{30} = -3.05$$

$$x_2 = 1037.5 \rightsquigarrow z_2 = \frac{1037.5 - 1000}{30} = \frac{37.5}{30} = 1.25$$

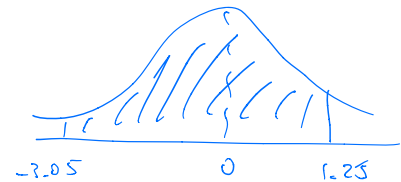
$$P(908.5 \leq X \leq 1037.5) \approx P(-3.05 \leq Z \leq 1.25)$$

$$= A(1.25) + A(3.05)$$

$$= 0.3944 + 0.4989$$

$$= 0.8933$$

$$= 89.33\%$$



13. (10 pts.) Wolfgang wants to buy some boxes of chocolates and some bags of chocolates. Each box costs \$15, contains 30 chocolates, and weighs 2 pounds. Each bag costs \$18, contains 20 (fancier) chocolates, and weighs 3 pounds. He wants the total number of chocolates to be at least 150, and he wants the total weight to be at most 40 pounds. Let x be the number of boxes that he buys and let y be the number of bags that he buys. He wants to choose x and y so that he minimizes the amount of money that he spends.

- (a) Find the objective function for this linear programming problem.

$$z = 15x + 18y$$

- (b) Do we want to maximize or minimize the objective function?

minimize

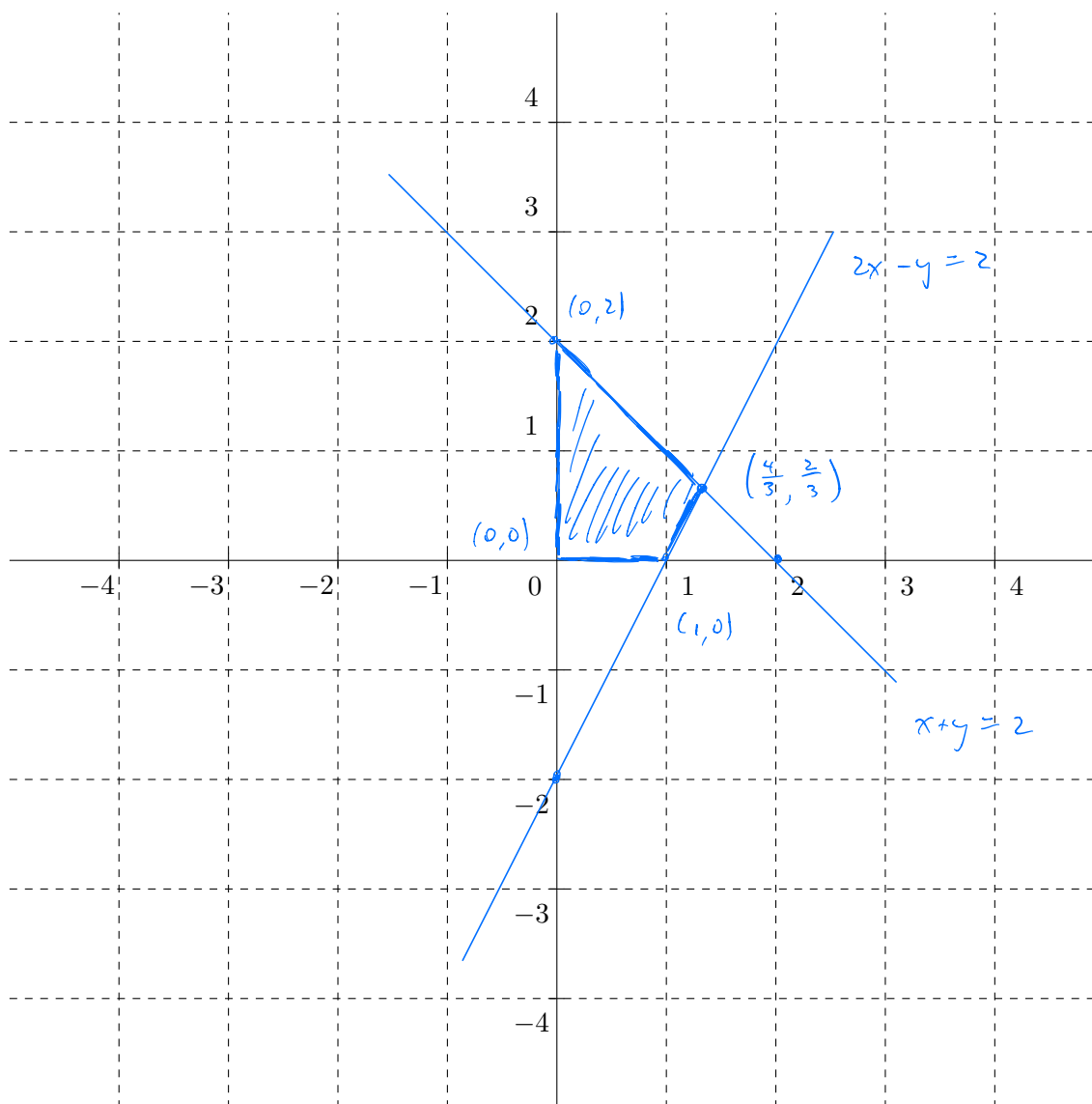
- (c) Write the constraints (inequalities) for this problem. **(You do NOT have to solve the corresponding optimization problem. Stop after you write the constraints.)**

number of chocolates	$30x + 20y \geq 150$
number of pounds	$2x + 3y \leq 40$
	$x \geq 0, y \geq 0$

14. (10 pts.) Consider the following system of inequalities:

$$\begin{aligned} 2x - y &\leq 2 \\ x + y &\leq 2 \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

Sketch the feasible set using the following axes. **Be sure to label the lines**, shade the feasible set, and label *all* the corners of the feasible set.



$$\begin{aligned} 2x - y &= 2 \\ x + y &= 2 \\ \hline 3x &= 4 \\ x &= \frac{4}{3} \end{aligned}$$

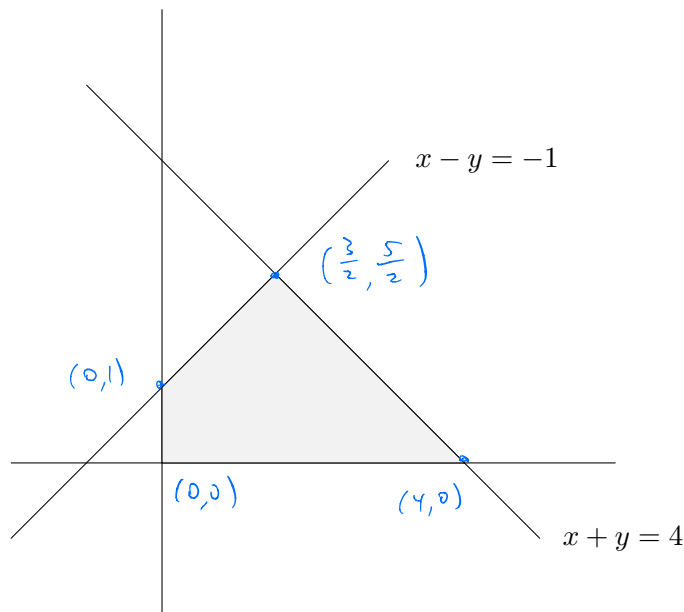
$$y = \frac{2}{3}$$

15. (10 pts.)

Consider the constraints

$$\begin{aligned}x - y &\geq -1 \\x + y &\leq 4 \\x &\geq 0, y \geq 0\end{aligned}$$

The following is a sketch of the feasible region.



(a) Find the coordinates of the corners of the feasible region and label the picture accordingly.

$$\begin{aligned}x - y &= -1 \\x + y &= 4 \\ \hline 2x &= 3 \\x &= \frac{3}{2} \quad y = \frac{5}{2}\end{aligned}$$

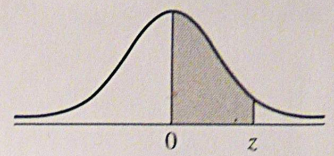
(b) What is the maximum possible value of the objective function $z = 2x + 3y$?

$$\begin{aligned}(0, 0) &\rightsquigarrow 2(0) + 3(0) = 0 \\(4, 0) &\rightsquigarrow 2(4) + 3(0) = 8 \\(\frac{3}{2}, \frac{5}{2}) &\rightsquigarrow 2(\frac{3}{2}) + 3(\frac{5}{2}) = \frac{6}{2} + \frac{15}{2} = \frac{21}{2} = 10.5 \\(0, 1) &\rightsquigarrow 2(0) + 3(1) = 3\end{aligned}$$

(c) What values of x and y maximize the objective function $z = 2x + 3y$?

$$x = \frac{3}{2}, \quad y = \frac{5}{2}$$

Area Under the Standard Normal Curve



z	A	z	A	z	A	z	A	z	A	z	A	z	A
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332	2.00	0.4773	2.50	0.4938	3.00	0.4987
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345	2.01	0.4778	2.51	0.4940	3.01	0.4987
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357	2.02	0.4783	2.52	0.4941	3.02	0.4987
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370	2.03	0.4788	2.53	0.4943	3.03	0.4988
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382	2.04	0.4793	2.54	0.4945	3.04	0.4988
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394	2.05	0.4798	2.55	0.4946	3.05	0.4989
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406	2.06	0.4803	2.56	0.4948	3.06	0.4989
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418	2.07	0.4808	2.57	0.4949	3.07	0.4989
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4430	2.08	0.4812	2.58	0.4951	3.08	0.4990
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441	2.09	0.4817	2.59	0.4952	3.09	0.4990
0.10	0.0398	0.60	0.2258	1.10	0.3643	1.60	0.4452	2.10	0.4821	2.60	0.4953	3.10	0.4990
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463	2.11	0.4826	2.61	0.4955	3.11	0.4991
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474	2.12	0.4830	2.62	0.4956	3.12	0.4991
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4485	2.13	0.4834	2.63	0.4957	3.13	0.4991
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495	2.14	0.4838	2.64	0.4959	3.14	0.4992
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505	2.15	0.4842	2.65	0.4960	3.15	0.4992
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515	2.16	0.4846	2.66	0.4961	3.16	0.4992
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525	2.17	0.4850	2.67	0.4962	3.17	0.4992
0.18	0.0714	0.68	0.2518	1.18	0.3810	1.68	0.4535	2.18	0.4854	2.68	0.4963	3.18	0.4993
0.19	0.0754	0.69	0.2549	1.19	0.3830	1.69	0.4545	2.19	0.4857	2.69	0.4964	3.19	0.4993
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554	2.20	0.4861	2.70	0.4965		
0.21	0.0832	0.71	0.2612	1.21	0.3869	1.71	0.4564	2.21	0.4865	2.71	0.4966		
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573	2.22	0.4868	2.72	0.4967		
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582	2.23	0.4871	2.73	0.4968		
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591	2.24	0.4875	2.74	0.4969		
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599	2.25	0.4878	2.75	0.4970		
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608	2.26	0.4881	2.76	0.4971		
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616	2.27	0.4884	2.77	0.4972		
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625	2.28	0.4887	2.78	0.4973		
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633	2.29	0.4890	2.79	0.4974		
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641	2.30	0.4893	2.80	0.4974		
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649	2.31	0.4896	2.81	0.4975		
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656	2.32	0.4898	2.82	0.4976		
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664	2.33	0.4901	2.83	0.4977		
0.34	0.1331	0.84	0.2996	1.34	0.4099	1.84	0.4671	2.34	0.4904	2.84	0.4977		
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678	2.35	0.4906	2.85	0.4978		
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686	2.36	0.4909	2.86	0.4979		
0.37	0.1443	0.87	0.3079	1.37	0.4147	1.87	0.4693	2.37	0.4911	2.87	0.4980		
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4700	2.38	0.4913	2.88	0.4980		
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706	2.39	0.4916	2.89	0.4981		
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713	2.40	0.4918	2.90	0.4981		
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719	2.41	0.4920	2.91	0.4982		
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726	2.42	0.4922	2.92	0.4983		
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732	2.43	0.4925	2.93	0.4983		
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738	2.44	0.4927	2.94	0.4984		
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744	2.45	0.4929	2.95	0.4984		
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750	2.46	0.4931	2.96	0.4985		
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756	2.47	0.4932	2.97	0.4985		
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4762	2.48	0.4934	2.98	0.4986		
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767	2.49	0.4936	2.99	0.4986		

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- | | | | | | |
|-----|------------------|------------------|------------------|------------------|------------------|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. _____

11. _____

12. _____

13. _____

14. _____

15. _____

Tot. _____