

### Exam 3

April 18, 2019

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**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.** The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (e) |
| 2.  | (a) | (b) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (e) |
| 4.  | (a) | (b) | (c) | (d) | (e) |
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| 6.  | (a) | (b) | (c) | (d) | (e) |
| 7.  | (a) | (b) | (c) | (d) | (e) |
| 8.  | (a) | (b) | (c) | (d) | (e) |
| 9.  | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) The 50 students in Math 456 took a 10-point quiz, and the following gives the number of students scoring 10, 9, 8, 7 and 6. (No other scores were obtained.)

score	10	9	8	7	6
number of students	22	12	6	7	3

Find the **relative frequency** of the score “10.”

- (a) 0.22            (b) 0.44            (c) 22            (d) 0.2            (e) 0.1

2. (5 pts.) There are six students shooting baskets. The first five made 10, 8, 11, 6 and 13 baskets, respectively. How many baskets must the sixth student make in order for the average (mean) number for all six to be 10?

- (a) 10            (b) 11            (c) 12            (d) 13            (e) 14

3. (5 pts.) Find the population variance for the following set of data (up to 2 decimal places). [Note that this is asking for the variance, **not** the standard deviation.]

11, 13, 8, 14, 9.

- (a) 4.8                      (b) 0                      (c) 2.28                      (d) 26                      (e) 5.2

4. (5 pts.) A box contains 8 blue balls, 6 red balls and 4 green balls, all mixed together. A ball is selected randomly from the urn. If it is green, you stop. If it is not green, you throw it away (NOT back in the urn) and select another one randomly. This is repeated until you select a green ball. Let  $X$  denote the random variable that gives the number of times you selected a ball. (For example, if you didn't get a green ball on the first selection but did on the second selection then  $X = 2$  because you drew a total of two balls.) What values may  $X$  assume?

- (a)  $\{1, 2, 3, \dots, 15\}$                       (b)  $\{2, 3, \dots, 15\}$                       (c)  $\{1, 2, 3, \dots, 14\}$   
(d)  $\{1, 2, 3, \dots, 18\}$                       (e)  $\{2, 3, \dots, 14\}$

5. (5 pts.) An urn contains three balls marked “1,” three balls marked “2” and three balls marked “3.” Claire selects two balls at random. Let  $X$  be the sum of the numbers on the two selected balls. Find the probability distribution for  $X$ . [Hint: notice that  $4 = 1 + 3 = 2 + 2$ .]

(a) 

$x$	$P(x)$
2	$2/36$
3	$8/36$
4	$16/36$
5	$8/36$
6	$2/36$

(b) 

$x$	$P(x)$
2	$4/36$
3	$9/36$
4	$10/36$
5	$9/36$
6	$4/36$

(c) 

$x$	$P(x)$
2	$3/36$
3	$8/36$
4	$14/36$
5	$8/36$
6	$3/36$

(d) 

$x$	$P(x)$
2	$4/36$
3	$8/36$
4	$12/36$
5	$8/36$
6	$4/36$

(e) 

$x$	$P(x)$
2	$3/36$
3	$9/36$
4	$12/36$
5	$9/36$
6	$3/36$

6. (5 pts.) Compute the **standard deviation**  $\sigma(X)$  for the random variable defined as follows (to two decimal places):

$x_i$	$p_i$
10	0.3
20	0.2
30	0.5

(a) 8.72

(b) 22

(c) 0

(d) 8.84

(e) 8.93

7. (5 pts.) Some data has a mean of  $\mu = 32$  and standard deviation  $\sigma = 2.5$ . Find the  $z$ -score for a score of 28 (up to 2 decimal places).

- (a) 1.6                      (b) -4                      (c) -2                      (d) -1.6                      (e) 2

8. (5 pts.) A multiple choice exam has 10 questions, each containing 5 possible answers. Lee randomly guesses all the answers. You fail if you get 0 to 7 right and pass if you get 8 to 10 right. What is the probability that Lee will pass the exam?

- (a)  $\left(\frac{1}{5}\right)^8 \left(\frac{4}{5}\right)^2 + \left(\frac{1}{5}\right)^9 \left(\frac{4}{5}\right)^1 + \left(\frac{1}{5}\right)^{10} \left(\frac{4}{5}\right)^0$
- (b)  $C(10, 8) \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + C(10, 9) \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + C(10, 10) \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$
- (c)  $C(10, 8) \left(\frac{1}{5}\right)^8 \left(\frac{4}{5}\right)^2 + C(10, 9) \left(\frac{1}{5}\right)^9 \left(\frac{4}{5}\right)^1 + C(10, 10) \left(\frac{1}{5}\right)^{10} \left(\frac{4}{5}\right)^0$
- (d)  $\left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$
- (e)  $C(10, 8) \left(\frac{4}{5}\right)^8 \left(\frac{1}{5}\right)^2 + C(10, 9) \left(\frac{4}{5}\right)^9 \left(\frac{1}{5}\right)^1 + C(10, 10) \left(\frac{4}{5}\right)^{10} \left(\frac{1}{5}\right)^0$

9. (5 pts.) The weights of the members of some population are normally distributed with a mean of 30 lbs and a standard deviation of 8 lbs. Let  $X$  be the random variable corresponding to this normal distribution. If a member of that population is chosen at random, find  $P(28 \leq X \leq 32)$ .

- (a) 9.87%      (b) 98.76%      (c) 49.38%      (d) 19.74%      (e) 38.30%

10. (5 pts.) Which of the following points is in the feasible region for the following system of inequalities? (Be careful with  $\leq$  versus  $<$  and with  $\geq$  versus  $>$ .)

$$\begin{aligned}2x + 3y &\geq 4 \\5x - 6y &< 4 \\x \geq 0, \quad y &\geq 0\end{aligned}$$

- (a) (2, 1)      (b) (1, 1)      (c) (-1, 2)      (d) (0, 1)      (e) (2, 0)

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

**11.** (10 pts.) A shelf contains 10 CDs, of which 6 are Beatles and 4 are Simon and Garfunkel. These CDs are all mixed together, and Emily randomly chooses two to take with her on a trip. Let  $X$  be the random variable counting **the number of Beatles CDs that she picks**.

(a) What are the possible values that  $X$  may take?

(b) Find the probability distribution for  $X$ .

(c) What is the expected value for this probability distribution? [It is not necessarily going to be an integer.]

**12.** (10 pts.) Exactly 10% of a certain population own a Harry Potter book. Suppose that 10,000 people are chosen at random and asked if they own a Harry Potter book.

[Note: the formulas

$$\begin{aligned}\mu &= np \\ \sigma &= \sqrt{npq}\end{aligned}$$

may be useful in this problem.]

(a) (3 points) Find the mean and standard deviation of this binomial distribution.

(b) (7 points) Use the normal approximation to the binomial distribution to estimate the probability that between 909 and 1037 of the 10,000 people that were randomly chosen own a Harry Potter book.

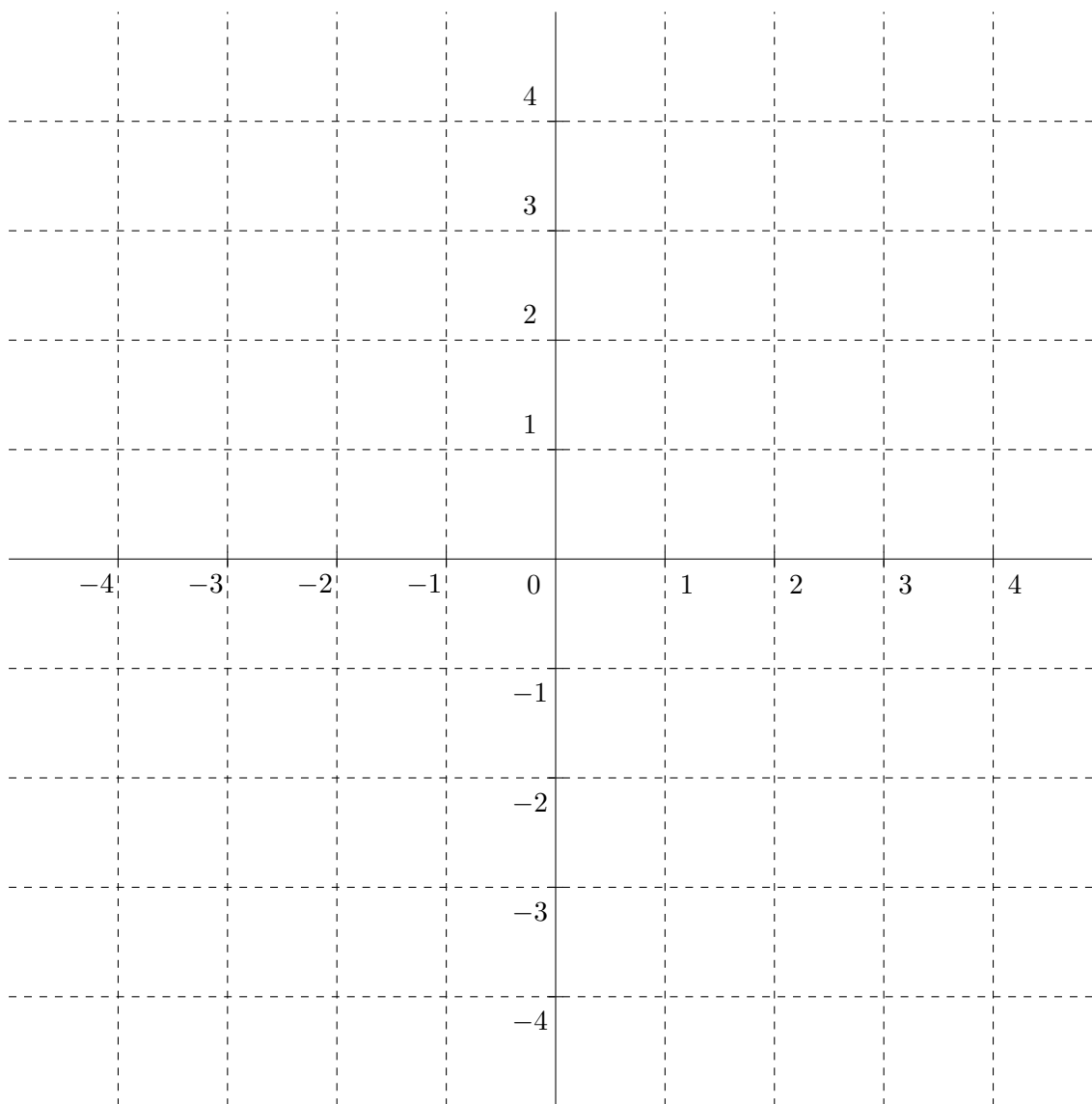




14. (10 pts.) Consider the following system of inequalities:

$$\begin{aligned} 2x - y &\leq 2 \\ x + y &\leq 2 \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

Sketch the feasible set using the following axes. **Be sure to label the lines**, shade the feasible set, and label *all* the corners of the feasible set.

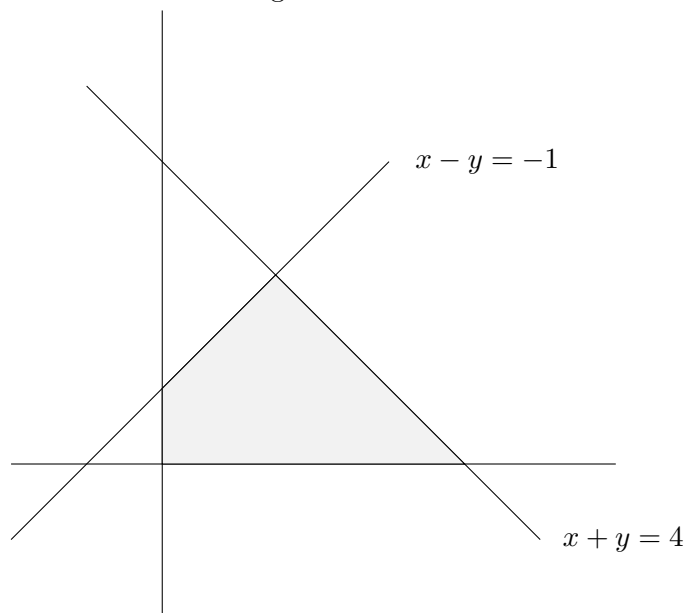


15. (10 pts.)

Consider the constraints

$$\begin{aligned}x - y &\geq -1 \\x + y &\leq 4 \\x &\geq 0, y \geq 0\end{aligned}$$

The following is a sketch of the feasible region.

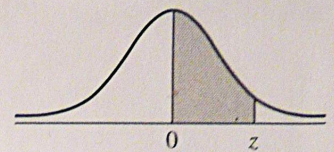


(a) Find the coordinates of the corners of the feasible region and label the picture accordingly.

(b) What is the maximum possible value of the objective function  $z = 2x + 3y$ ?

(c) What values of  $x$  and  $y$  maximize the objective function  $z = 2x + 3y$ ?

# Area Under the Standard Normal Curve



$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332	2.00	0.4773	2.50	0.4938	3.00	0.4987
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345	2.01	0.4778	2.51	0.4940	3.01	0.4987
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357	2.02	0.4783	2.52	0.4941	3.02	0.4987
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370	2.03	0.4788	2.53	0.4943	3.03	0.4988
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382	2.04	0.4793	2.54	0.4945	3.04	0.4988
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394	2.05	0.4798	2.55	0.4946	3.05	0.4989
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406	2.06	0.4803	2.56	0.4948	3.06	0.4989
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418	2.07	0.4808	2.57	0.4949	3.07	0.4989
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4430	2.08	0.4812	2.58	0.4951	3.08	0.4990
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441	2.09	0.4817	2.59	0.4952	3.09	0.4990
0.10	0.0398	0.60	0.2258	1.10	0.3643	1.60	0.4452	2.10	0.4821	2.60	0.4953	3.10	0.4990
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463	2.11	0.4826	2.61	0.4955	3.11	0.4991
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474	2.12	0.4830	2.62	0.4956	3.12	0.4991
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4485	2.13	0.4834	2.63	0.4957	3.13	0.4991
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495	2.14	0.4838	2.64	0.4959	3.14	0.4992
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505	2.15	0.4842	2.65	0.4960	3.15	0.4992
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515	2.16	0.4846	2.66	0.4961	3.16	0.4992
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525	2.17	0.4850	2.67	0.4962	3.17	0.4992
0.18	0.0714	0.68	0.2518	1.18	0.3810	1.68	0.4535	2.18	0.4854	2.68	0.4963	3.18	0.4993
0.19	0.0754	0.69	0.2549	1.19	0.3830	1.69	0.4545	2.19	0.4857	2.69	0.4964	3.19	0.4993
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554	2.20	0.4861	2.70	0.4965		
0.21	0.0832	0.71	0.2612	1.21	0.3869	1.71	0.4564	2.21	0.4865	2.71	0.4966		
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573	2.22	0.4868	2.72	0.4967		
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582	2.23	0.4871	2.73	0.4968		
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591	2.24	0.4875	2.74	0.4969		
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599	2.25	0.4878	2.75	0.4970		
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608	2.26	0.4881	2.76	0.4971		
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616	2.27	0.4884	2.77	0.4972		
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625	2.28	0.4887	2.78	0.4973		
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633	2.29	0.4890	2.79	0.4974		
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641	2.30	0.4893	2.80	0.4974		
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649	2.31	0.4896	2.81	0.4975		
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656	2.32	0.4898	2.82	0.4976		
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664	2.33	0.4901	2.83	0.4977		
0.34	0.1331	0.84	0.2996	1.34	0.4099	1.84	0.4671	2.34	0.4904	2.84	0.4977		
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678	2.35	0.4906	2.85	0.4978		
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686	2.36	0.4909	2.86	0.4979		
0.37	0.1443	0.87	0.3079	1.37	0.4147	1.87	0.4693	2.37	0.4911	2.87	0.4980		
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4700	2.38	0.4913	2.88	0.4980		
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706	2.39	0.4916	2.89	0.4981		
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713	2.40	0.4918	2.90	0.4981		
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719	2.41	0.4920	2.91	0.4982		
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726	2.42	0.4922	2.92	0.4983		
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732	2.43	0.4925	2.93	0.4983		
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738	2.44	0.4927	2.94	0.4984		
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744	2.45	0.4929	2.95	0.4984		
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750	2.46	0.4931	2.96	0.4985		
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756	2.47	0.4932	2.97	0.4985		
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4762	2.48	0.4934	2.98	0.4986		
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767	2.49	0.4936	2.99	0.4986		

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| 2.  | (a)              | (b)              | ( <del>c</del> ) | (d)              | (e)              |
| 3.  | (a)              | (b)              | (c)              | (d)              | ( <del>e</del> ) |
| 4.  | ( <del>a</del> ) | (b)              | (c)              | (d)              | (e)              |
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| 6.  | ( <del>a</del> ) | (b)              | (c)              | (d)              | (e)              |
| 7.  | (a)              | (b)              | (c)              | ( <del>d</del> ) | (e)              |
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| 9.  | (a)              | (b)              | (c)              | ( <del>d</del> ) | (e)              |
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MC. \_\_\_\_\_

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12. \_\_\_\_\_

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Tot. \_\_\_\_\_