

**Math 10120 Finite Math**  
**Practice Final Exam 1**  
**May 8, 2019**

Name: Solutions

- Be sure that you have all 18 pages of the test.
- The exam lasts for 2 hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

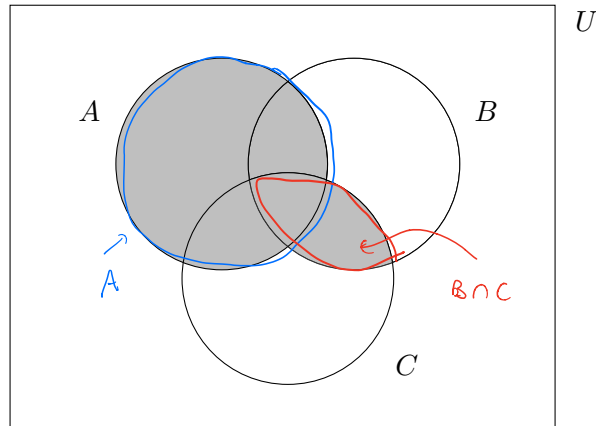
PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                                    |
|-------------------------|------------------------------------|
| 1. (a) (b) (c) (d) (e)  | 17. (a) (b) (c) (d) (e)            |
| 2. (a) (b) (c) (d) (e)  | 18. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 3. (a) (b) (c) (d) (e)  | 19. (a) (b) (c) (d) (e)            |
| 4. (a) (b) (c) (d) (e)  | 20. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 5. (a) (b) (c) (d) (e)  | 21. (a) (b) (c) (d) (e)            |
| 6. (a) (b) (c) (d) (e)  | 22. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 7. (a) (b) (c) (d) (e)  | 23. (a) (b) (c) (d) (e)            |
| 8. (a) (b) (c) (d) (e)  | 24. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 9. (a) (b) (c) (d) (e)  | 25. (a) (b) (c) (d) (e)            |
| 10. (a) (b) (c) (d) (e) | 26. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 11. (a) (b) (c) (d) (e) | 27. (a) (b) (c) (d) (e)            |
| 12. (a) (b) (c) (d) (e) | 28. (a) (b) (c) (d) (e)            |
| .....                   |                                    |
| 13. (a) (b) (c) (d) (e) | <del>29. (a) (b) (c) (d) (e)</del> |
| 14. (a) (b) (c) (d) (e) | <del>20. (a) (b) (c) (d) (e)</del> |
| .....                   |                                    |
| 15. (a) (b) (c) (d) (e) |                                    |
| 16. (a) (b) (c) (d) (e) |                                    |

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Multiple Choice

1. (5 pts.) Which of the following corresponds to the area shaded in gray in the following Venn diagram? Remember that the notation  $( )'$  refers to the complement of the set  $( )$ .



- (a)  $A \cup (B \cap C)$      
 (b)  $(B \cup C)' \cap A$      
 (c)  $(A \cap B) \cup (A \cap C) \cup (B \cap C)$   
 (d)  $A \cap (B \cup C)$      
 (e)  $A \cup (B \cap C)'$

2. (5 pts.) Let  $M$  be the set of Notre Dame students enrolled in a math class, let  $B$  be the set of Notre Dame students enrolled in a biology class, and let  $E$  be the set of Notre Dame students enrolled in an engineering class. Let the universal set,  $U$ , be the set of all Notre Dame students. In words, what is  $(M \cap B') \cap E$ ? [Feel free to draw a Venn diagram if it helps.]

- (a) Notre Dame students that are enrolled in math, engineering **and** biology  
 (b) Notre Dame students that are enrolled in engineering, **and either** enrolled in math **or not** enrolled in biology  
 (c) Notre Dame students that are **either** enrolled in math **or** enrolled in engineering, but **not** enrolled in biology  
 (d) Notre Dame students that are **not** enrolled in engineering **or** math **or** biology  
 (e) Notre Dame students that are enrolled in math **and** engineering but **not** biology.

$\cap$  = "and"

$M \cap B'$  = in M and not in B

Put it all together:

in M and in E but not in B

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3. (5 pts.) As they cross the mountains out of Austria, Captain Von Trapp decides to take a picture of his wife, Maria, and their four daughters and two sons. He insists that the four daughters have to stand together and the two sons have to stand together. His wife can either be on the far left, or between the boys and the girls, or on the far right. In how many ways can they line up to take the picture?

- (a) 144      ~~(b)~~ 288      (c) 48      (d) 24      (e) 96

Possibilities (S=sons, D=daughters, W=wife)

SDW      In each case there are  $2!$  ways to line up the sons and  $4!$  ways  
WSD      to line up the daughters, for a total of  
DSW  
WDS       $6(2!)(4!) = 6 \cdot 48 = 288$   
SWD  
DWS

4. (5 pts.) How many different words (including nonsense words) can be made from rearrangements of the letters in the word

B A S K E T B A L L?

(Note there are two B's, two A's and two L's. The "words" must use all 10 letters.)

- (a)  $10!$       ~~(b)~~  $\frac{10!}{(2!)^3}$       (c)  $\frac{10!}{3! \cdot 7!}$       (d)  $\frac{10!}{3!}$       (e)  $\frac{10!}{(3!)^2}$

10 letters total with repeats of B, A and L (two each)

$$\frac{10!}{2! \cdot 2! \cdot 2!}$$

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5. (5 pts.) A club has 11 members. They have to choose some members to form a planning committee for an upcoming event. The conditions are:

- (i) At least one club member has to be on the planning committee.
- (ii) Bob, as club president, is not going to be on the committee.

In how many ways can they choose the committee?

- (a)  $2^{11} - 2$       (b)  $2^{11} - 1$       (c)  $2^{10}$       ~~(d)~~  $2^{10} - 1$       (e)  $2^{10} - 2$

Bob is removed, so there are 10 members in question.

You want the total number of subsets except the empty set.

$$2^{10} - 1$$

6. (5 pts.) Alice, Bob, Connie and Doug have just finished their main course at a restaurant, and the waiter brings a tray with 8 different pieces of (different kinds of) cake. In how many ways can the waiter give a piece of cake from the tray to each of Alice, Bob, Connie and Doug? [E.g. if he gives the piece of chocolate cake to Alice, he can't also give it to Bob, Connie or Doug.]

- ~~(a)~~ 1680      (b) 70      (c) 336      (d) 32      (e) 4096

$$P(8, 4) = \frac{8}{A} \frac{7}{B} \frac{6}{C} \frac{5}{D} = 1680$$

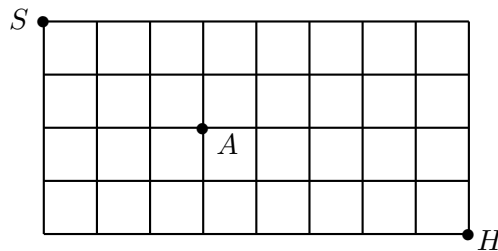
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7. (5 pts.) A litter of puppies has six male puppies and three female puppies. In how many ways can little Billy choose three so that he takes two males and one female?

- (a) 84                      (b) 18                      ~~(c) 45~~                      (d) 90                      (e) 36

$$C(6, 2) \cdot C(3, 1) = 15 \cdot 3 = 45$$

8. (5 pts.) Rafael has to walk 12 blocks, from the school (marked by  $S$  on the following map) to his house ( $H$ ), but on the way he wants to stop by his friend Adela's house ( $A$ ). How many possible routes does he have to choose from, assuming that he only walks east or south?



- ~~(a) 210~~                      (b) 15                      (c) 30                      (d) 35                      (e) 105

$$\begin{aligned} \# \text{ routes from } S \text{ to } A &: 5 \text{ blocks, } 2 \text{ south} & C(5, 2) = 10 \\ \# \text{ routes from } A \text{ to } H &: 7 \text{ blocks, } 2 \text{ south} & C(7, 2) = 21 \end{aligned}$$

$$10 \cdot 21 = 210$$

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9. (5 pts.) An experiment has sample space  $S = \{a, b, c, d, e\}$ . Define events  $E = \{a, b\}$  and  $F = \{b, d\}$ . If  $P(c) = 0.2$  and  $P(e) = 0.4$ , what is  $P(E \cup F)$ ?

- (a) 0.6                      ~~(b)~~ 0.4                      (c) 1  
(d) 0                      (e) There is not enough information to say.

$$\begin{aligned} P(E \cup F) &= P(\{a, b, d\}) \\ &= 1 - P(\{a, b, d\}^c) \\ &= 1 - P(\{c, e\}) \\ &= 1 - 0.2 - 0.4 \end{aligned}$$

$\begin{aligned} &= 1 - 0.6 \\ &= 0.4 \end{aligned}$

10. (5 pts.) I place some of my dog's toys in a box, and I tell him to choose one toy. The box contains 4 tennis balls, 2 orange rubber balls, and 1 flying disc. The orange rubber balls are his favorite toy, so he tends to choose the orange balls more often than the other toys. If  $S$  is the sample space,  $T$  is the event my dog chooses a tennis ball, and  $O$  is the event my dog chooses an orange ball, which of the following is FALSE?

- (a)  $T$  and  $O$  are mutually exclusive events  
(b)  $P(S) = 1$   
(c)  $n(T \cup O) = 6$   
(d)  $T \subseteq S$   
~~(e)~~ The outcomes in  $S$  are equally likely

It says that the dog chooses the orange balls more often than the other toys.

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11. (5 pts.) If  $E$  and  $F$  are events,  $P(E) = 0.3$ ,  $P(F) = 0.5$ , and  $P(E \cup F) = 0.6$ , what is  $P(E \cap F)$ ?

- (a) 0.6      (b) 0      (c) 0.8      ~~(d) 0.2~~      (e) 0.15

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$0.6 = 0.3 + 0.5 - P(E \cap F)$$

$$P(E \cap F) = 0.3 + 0.5 - 0.6$$

$$= 0.2$$

12. (5 pts.) An experiment consists of rolling a 6-sided die twice and recording the two numbers rolled. What is the probability the same number is rolled on both dice?

- (a)  $\frac{1}{36}$       (b)  $\frac{1}{18}$       ~~(c)  $\frac{1}{6}$~~       (d)  $\frac{1}{3}$       (e)  $\frac{5}{6}$

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

36 possible outcomes  
we want  $(1,1), (2,2), \dots, (6,6)$  so  $\frac{6}{36} = \frac{1}{6}$

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13. (5 pts.) Calvin draws 5 cards from a standard deck of 52 cards. What is the probability he draws 3 Queens and 2 aces?

- (a)  $C(4,3)C(4,2)$       ~~(b)~~  $\frac{C(4,3)C(4,2)}{C(52,5)}$       (c)  $\frac{3 \cdot 2}{5}$   
 (d)  $\frac{C(4,3)}{C(52,5)}$       (e)  $\frac{C(52,3)C(49,2)}{C(52,5)}$

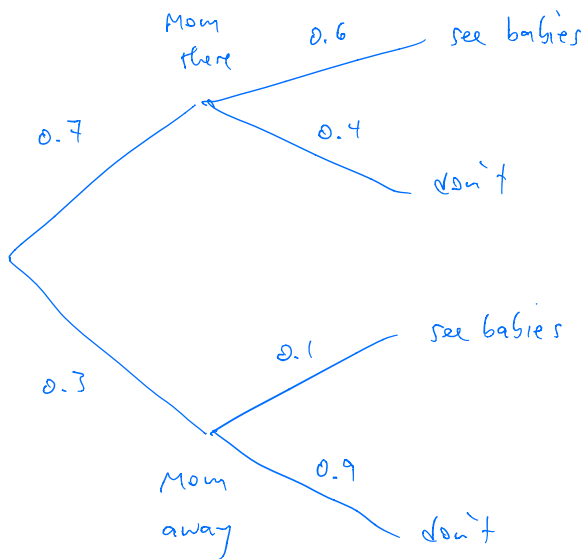
denominator is  $C(52,5)$ , the # of ways to choose 5 cards from a deck of 52.

# of ways to get 3 queens (out of 4) =  $C(4,3)$

" " " " 2 aces " " " =  $C(4,2)$

14. (5 pts.) Every summer, the math department courtyard is home to some (very cute) baby ducks. When the mother duck is away, I only have a 10% chance of seeing the baby ducks when I look out the window. When the mother duck is around, I have a 60% chance of seeing the baby ducks. The mother duck is around 70% of the time. If I look out the window, what is the probability I'll see the baby ducks? (Hint: A tree diagram may help!)

- ~~(a)~~ 0.45      (b) 0.1      (c) 0.6      (d) 0.7      (e) 0.42



$$\begin{aligned}
 P(\text{see babies}) &= (0.7)(0.6) + (0.3)(0.1) \\
 &= 0.42 + 0.03 \\
 &= 0.45
 \end{aligned}$$



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15. (5 pts.) The following table gives information about the books at a certain bookstore.

	Number of Paperbacks	Number of Hardcover
<b>Fiction</b>	4000	2000
<b>Non-fiction</b>	1000	4000

A book is chosen at random. What is the probability it is Fiction given that it is a paperback?

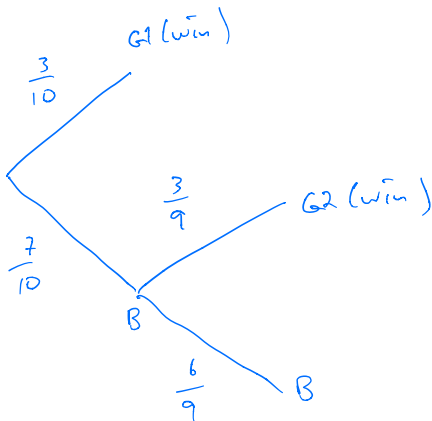
- (a)  $\frac{4}{11}$       (b)  $\frac{2}{3}$       (c)  ~~$\frac{4}{5}$~~       (d) 1      (e) 0

There are 5000 paperbacks, of which 4000 are fiction

$$\frac{4000}{5000} = \frac{4}{5}$$

16. (5 pts.) A bag contains 10 marbles, of which 7 are blue and 3 are green. Kevin plays a game where he draws up to 2 marbles (in succession and without replacement) from the bag. Once he draws a green marble, he stops, and he wins. If he doesn't have a green marble after he draws the second marble, he stops and he loses. If we know Kevin won the game, what is the probability the first marble Kevin drew was green?

- (a)  $\frac{3}{10}$       (b)  $\frac{8}{15}$       (c) 1      (d)  $\frac{7}{30}$       (e)  ~~$\frac{9}{16}$~~



$$\begin{aligned}
 P(G1 | W) &= \frac{P(G1 \cap W)}{P(W)} \\
 &= \frac{\left(\frac{3}{10}\right)}{\left(\frac{3}{10}\right) + \left(\frac{7}{10}\right)\left(\frac{3}{9}\right)} = \frac{\left(\frac{3}{10}\right)}{\left(\frac{27 + 21}{90}\right)} \\
 &= \left(\frac{3}{10}\right) \cdot \left(\frac{90}{48}\right) = \frac{9}{16}
 \end{aligned}$$

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17. (5 pts.) There are two printers which are convenient for me to use from my office. Printer A is out of order  $1/6$ th of the time, and Printer B is out of order  $1/5$ th of the time. Assuming the printers break down independently, what is the probability that at least one printer is working when I try to print final exams?

- (a)  $\frac{1}{30}$       (b)  $\frac{2}{3}$       (c)  $\frac{5}{6}$       ~~(d)  $\frac{29}{30}$~~       (e)  $\frac{4}{5}$

$$1 - P(\text{both out of order}) = 1 - \left(\frac{1}{6}\right)\left(\frac{1}{5}\right) = \frac{29}{30}$$

18. (5 pts.) Twenty college students are asked how many final exams they have to take this semester. The results are given in the following frequency distribution

Number of Exams	Frequency
0	4
1	3
2	4
3	7
4	2

What is the (population) standard deviation of the number of exams taken?

- (a) 1.70      (b) 2      (c) 1.79      (d) 1.34      ~~(e) 1.30~~

$x_i$	$f_i$	$x_i f_i$	$x_i - \mu$	$(x_i - \mu)^2$	$f_i (x_i - \mu)^2$
0	4	0	-2	4	16
1	3	3	-1	1	3
2	4	8	0	0	0
3	7	21	1	1	7
4	2	8	2	4	8
	<u>20</u>	<u>40</u>			<u>34</u>

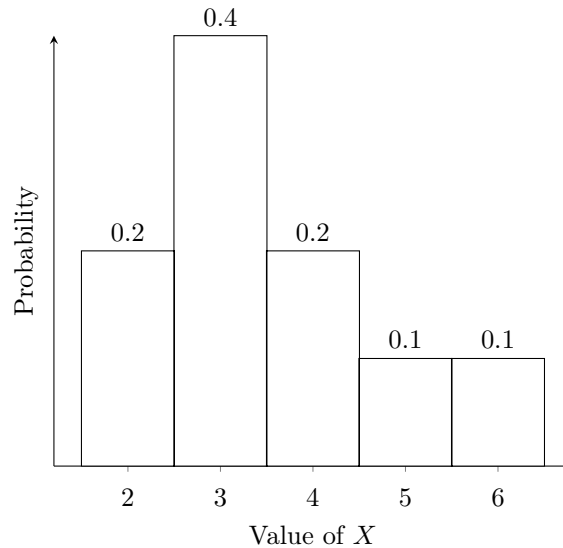
$$\mu = \frac{40}{20} = 2$$

$$\sigma^2 = \frac{34}{20} = 1.7$$

$$\sigma = \sqrt{1.7} = 1.30$$

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19. (5 pts.) We have the following histogram for a random variable,  $X$ .



What is  $P(X \leq 4)$ ?

- (a) 0.6       (b) 0.8      (c) 0.2      (d) 0.4      (e) 1

$$0.2 + 0.4 + 0.2 = 0.8$$

20. (5 pts.) Kelly pays \$2 to play the following game. Kelly draws two cards from a deck containing 20 cards consisting of 4 aces, 4 kings, 4 queens, 4 jacks, and 4 tens. She wins \$10 for each ace she draws and wins nothing if she draws no aces. Let  $X$  be Kelly's net earnings. Which of the following is the probability distribution for  $X$ ? (Note: We defined  $X$  to be Kelly's net earnings, so the possible values of  $X$  should take into account how much Kelly pays to play the game.)

(a)

$k$	$P(X = k)$
-2	12/19
8	32/95
18	3/95

(b)

$k$	$P(X = k)$
10	32/95
20	3/95

(c)

$k$	$P(X = k)$
8	32/95
18	3/95

(d)

$k$	$P(X = k)$
0	12/19
10	32/95
20	3/95

(e)

$k$	$P(X = k)$
-2	12/19
8	35/95

Note  
Even without computing probabilities, (a) is the only answer that accurately lists the possible outcomes (1st column)!!

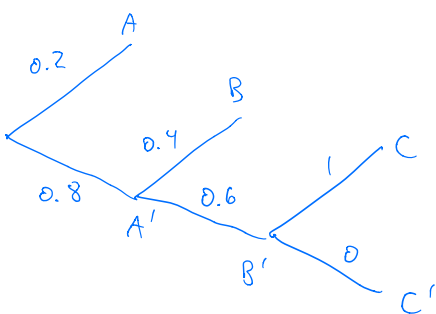
$k$	$P(X = k)$
-2	$C(16,2)/C(20,2) = 120/190 = 12/19$
8	$C(16,1)C(4,1)/C(20,2) = 64/190 = 32/95$
18	$C(4,2)/C(20,2) = 6/190 = 3/95$

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21. (5 pts.) Fred's flight home to Los Angeles for the summer has been canceled. He's booked on a new flight, flight C, that leaves 9 hours later. However, there are two earlier flights going to Los Angeles, and if there is space available, he can get on one of these flights instead. The first of the two, flight A, leaves in 2 hours, and there is a 20% chance Fred will be able to get on flight A. The next flight, flight B, leaves in 5 hours, and there is a 40% chance Fred will be able to get on flight B (assuming he hasn't already left on flight A). If Fred can't get on flights A or B, he will leave on flight C in 9 hours. Assuming Fred takes the earliest possible flight, how many hours (starting now) should Fred expect to be in the airport? (Hint: Define a random variable and compute its expected value. Keep in mind that Fred only takes flight B if he misses flight A and only takes flight C if he misses A and B.)

- (a) 6.32      (b) 9      (c) 11.4      (d) 4.6      (e) 5



$x_i$	$P_i$	$x_i P_i$
2	0.2	0.40
5	0.32	1.60
9	0.48	4.32
		<u>6.32</u>

22. (5 pts.) Hannah goes to the store every night during finals week to buy a Stayin' Awake energy drink. Unfortunately, everyone wants to buy a Stayin' Awake drink, so the store is often out. If the store only has Stayin' Awake energy drinks available about 30% of the time, what is the probability (rounded to 3 decimal places) Hannah will be able to buy a Stayin' Awake energy drink at least 2 of the 7 nights of finals week?

- (a) 0.329      (b) 0.753      (c) 0.996      (d) 0.671      (e) 0.004

$P(\text{success}) = p = 0.3$      $n = 7$     (success means she gets her drink)  
 $q = 0.7$

$$P(X \geq 2) = 1 - P(X = 0, 1)$$

$$= 1 - [C(7, 0) p^0 q^7 + C(7, 1) p^1 q^6]$$

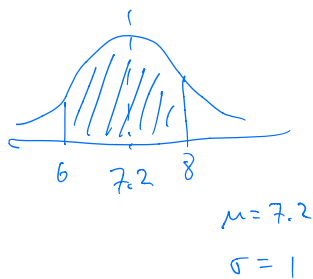
$$= 1 - [(0.7)^7 + 7(0.3)(0.7)^6] = 1 - 0.329$$

$$= 0.671$$

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23. (5 pts.) At a certain hospital, weights of newborn babies are normally distributed with mean 7.2 pounds and standard deviation 1 pound. What percentage of babies born at the hospital weigh between 6 and 8 pounds? (All answer choices are rounded to one decimal place.)

- (a) 2%                      (b) 78.8%                      ~~(c) 67.3%~~                      (d) 11.5%                      (e) 90.3%



$$z_1 = \frac{6 - 7.2}{1} = -1.2$$

$$z_2 = \frac{8 - 7.2}{1} = 0.8$$

$$P(6 \leq X \leq 8) = A(0.8) + A(1.2)$$

$$= 0.2881 + 0.3849$$

$$= 0.6730$$

$$= 67.3\%$$

24. (5 pts.) Consider the following linear programming problem:

A company has factories in Cleveland and in Toledo that manufacture sofas and wheelbarrows. Each day the Cleveland factory is operating, it produces 500 sofas and 400 wheelbarrows, at an operating cost of \$18,000. Each day the Toledo factory is operating, it produces 300 sofas and 200 wheelbarrows at an operating cost of \$15,000. An order is received for 2,500 sofas and 2,100 wheelbarrows. For how many days should each factory operate to fill the order at the least operating cost?

Which of the following is one of the constraints that we get when we translate the problem to mathematical language? In the following,  $x$  is the number of days the Cleveland factory should operate, and  $y$  is the number of days the Toledo factory should operate. [Hint: pay careful attention to the inequalities: should it be  $\leq$  or  $\geq$ ?]

- (a)  $400x + 200y \leq 2,100$                       ~~(b)  $400x + 200y \geq 2,100$~~
- (c)  $500x + 400y \leq 18,000$                       (d)  $500x + 400y \geq 2,100$
- (e)  $400x + 200y \leq 18,000$

sofas  $500x + 300y \geq 2500$

wheelbarrows  $400x + 200y \geq 2100$

$x \geq 0, y \geq 0$

cost  $C = 18000x + 15000y$  (objective function)  
- minimize

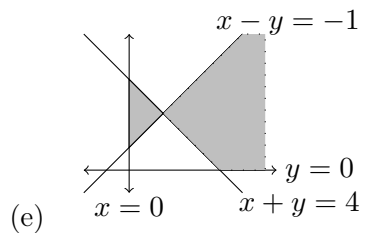
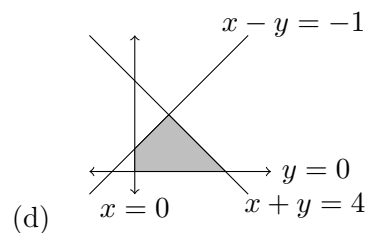
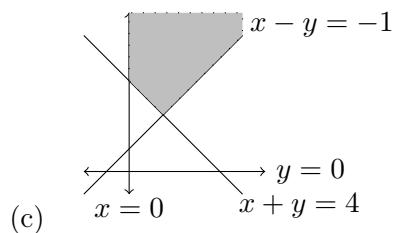
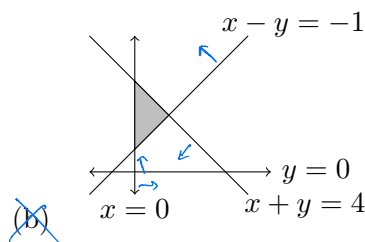
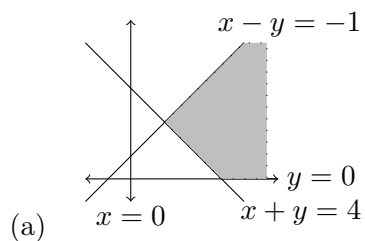
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25. (5 pts.) Consider the system of inequalities

$$\begin{aligned} x - y &\leq -1 \\ x + y &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Test point (0,0) fails since  $0 \leq -1$  is false  
 test point (0,0) works since  $0 \leq 4$  is true

Which of the following shaded regions is the feasible set for this system of linear inequalities?



26. (5 pts.) Find the following matrix product:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{aligned} [1 \ 2 \ 3] \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & -1 \end{bmatrix} &= [-1 \ 5] \\ [-1 \ 5] \begin{bmatrix} 3 \\ 1 \end{bmatrix} &= [2] \end{aligned}$$

~~(a)~~  $[2]$

(b)  $[-1 \ 5]$

(c)  $[20]$

(d)  $\begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

(e) This product does not exist.

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27. (5 pts.) In the 1980 presidential election, Reagan (R) and Carter (C) played a game where each one made a statement. If both made a true statement, Reagan picked up 10 percentage points (and Carter lost 10 percentage points) in the next poll. If Reagan's statement was true and Carter's was false, Reagan picked up 5 points. If Reagan's statement was false and Carter's was true, Carter picked up 3 points. If both statements were false, Reagan picked up 6 points. Which of the following is the pay-off matrix (for R) for this zero-sum game?

(a) 

	T	F
T	10	-3
F	5	6

(b) 

	T	F
T	-10	-5
F	3	-6

(c) 

	T	F
T	-10	3
F	-5	-6

(d) 

	T	F
T	10	6
F	-3	5

~~(e)~~

	T	F
T	10	5
F	-3	6

$$\begin{array}{c}
 \text{C} \\
 \text{T} \quad \text{F} \\
 \text{R} \begin{bmatrix} 10 & 5 \\ -3 & 6 \end{bmatrix}
 \end{array}$$

28. (5 pts.) The following is the pay-off matrix for the row player in a zero-sum game:

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ -3 & -2 & 5 \end{bmatrix}$$

Which of the following statements is true?

- (a) The game is strictly determined with a value of 5.
- (b) The game is strictly determined with a value of 3.
- ~~(c)~~ The game is strictly determined with a value of 1.
- (d) The game is strictly determined with a value of 0.
- (e) There is no saddle point in this matrix.

*R's calculation*

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ -3 & -2 & 5 \end{bmatrix}$$

*C's calculation*

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ -3 & -2 & 5 \end{bmatrix}$$

29. (5 pts.) Russia (R) and Canada (C) play a zero-sum game, with pay-off matrix for R given by

$$\begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix}.$$

What is Canada's optimal mixed strategy matrix? **Note:** the formula given at the end of the exam may help (page 17).

- (a)  $\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$       (b)  $\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$       (c)  $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$       (d)  $\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$       (e)  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix}$$

$$q = \frac{4-2}{5+4-2-1} = \frac{2}{6} = \frac{1}{3}$$

answer!  $\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$

30. (5 pts.) Rudolph (R) and Comet (C) play a zero-sum game with pay-off matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix}.$$

Which of the following are the strategy line(s) for Rudolph (where  $y$  is the expected pay-off)?

- (a)  $y = 4p + 1$  and  $y = 4 - 2p$       (b)  $y = 3p + 2$  and  $y = 4 - 3p$   
 (c)  $y = 6pq - 2p - 3q + 4$       (d)  $y = 5p + 1$  and  $y = 2p + 4$   
 (e) This is a fair game so there are no strategy lines.

ignore



Name: \_\_\_\_\_

### Game Theory Formulas.

For a mixed strategy game with payoff matrix

$$R \begin{matrix} & \begin{matrix} c_1 & c_2 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \end{matrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{matrix}$$

R's optimal strategy is given by  $[p_1 \ p_2]$ , where

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \text{ and } p_2 = 1 - p_1.$$

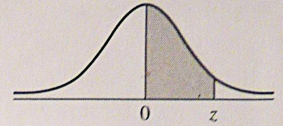
C's optimal strategy is given by  $\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ , where

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \text{ and } q_2 = 1 - q_1.$$

If both players play their optimal strategies, then the expected payoff (or value of the game) is

$$v = \frac{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}.$$

## Area Under the Standard Normal Curve



$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$	$z$	$A$
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332	2.00	0.4773	2.50	0.4938	3.00	0.4987
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345	2.01	0.4778	2.51	0.4940	3.01	0.4987
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357	2.02	0.4783	2.52	0.4941	3.02	0.4987
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370	2.03	0.4788	2.53	0.4943	3.03	0.4988
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382	2.04	0.4793	2.54	0.4945	3.04	0.4988
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394	2.05	0.4798	2.55	0.4946	3.05	0.4989
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406	2.06	0.4803	2.56	0.4948	3.06	0.4989
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418	2.07	0.4808	2.57	0.4949	3.07	0.4989
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4430	2.08	0.4812	2.58	0.4951	3.08	0.4990
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441	2.09	0.4817	2.59	0.4952	3.09	0.4990
0.10	0.0398	0.60	0.2258	1.10	0.3643	1.60	0.4452	2.10	0.4821	2.60	0.4953	3.10	0.4990
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463	2.11	0.4826	2.61	0.4955	3.11	0.4991
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474	2.12	0.4830	2.62	0.4956	3.12	0.4991
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4485	2.13	0.4834	2.63	0.4957	3.13	0.4991
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495	2.14	0.4838	2.64	0.4959	3.14	0.4992
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505	2.15	0.4842	2.65	0.4960	3.15	0.4992
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515	2.16	0.4846	2.66	0.4961	3.16	0.4992
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525	2.17	0.4850	2.67	0.4962	3.17	0.4992
0.18	0.0714	0.68	0.2518	1.18	0.3810	1.68	0.4535	2.18	0.4854	2.68	0.4963	3.18	0.4993
0.19	0.0754	0.69	0.2549	1.19	0.3830	1.69	0.4545	2.19	0.4857	2.69	0.4964	3.19	0.4993
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554	2.20	0.4861	2.70	0.4965		
0.21	0.0832	0.71	0.2612	1.21	0.3869	1.71	0.4564	2.21	0.4865	2.71	0.4966		
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573	2.22	0.4868	2.72	0.4967		
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582	2.23	0.4871	2.73	0.4968		
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591	2.24	0.4875	2.74	0.4969		
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599	2.25	0.4878	2.75	0.4970		
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608	2.26	0.4881	2.76	0.4971		
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616	2.27	0.4884	2.77	0.4972		
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625	2.28	0.4887	2.78	0.4973		
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633	2.29	0.4890	2.79	0.4974		
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641	2.30	0.4893	2.80	0.4974		
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649	2.31	0.4896	2.81	0.4975		
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656	2.32	0.4898	2.82	0.4976		
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664	2.33	0.4901	2.83	0.4977		
0.34	0.1331	0.84	0.2996	1.34	0.4099	1.84	0.4671	2.34	0.4904	2.84	0.4977		
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678	2.35	0.4906	2.85	0.4978		
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686	2.36	0.4909	2.86	0.4979		
0.37	0.1443	0.87	0.3079	1.37	0.4147	1.87	0.4693	2.37	0.4911	2.87	0.4980		
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4700	2.38	0.4913	2.88	0.4980		
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706	2.39	0.4916	2.89	0.4981		
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713	2.40	0.4918	2.90	0.4981		
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719	2.41	0.4920	2.91	0.4982		
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726	2.42	0.4922	2.92	0.4983		
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732	2.43	0.4925	2.93	0.4983		
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738	2.44	0.4927	2.94	0.4984		
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744	2.45	0.4929	2.95	0.4984		
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750	2.46	0.4931	2.96	0.4985		
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756	2.47	0.4932	2.97	0.4985		
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4762	2.48	0.4934	2.98	0.4986		
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767	2.49	0.4936	2.99	0.4986		

**Math 10120 Finite Math**  
**Practice Final Exam 1**  
**May 8, 2019**

Name: \_\_\_\_\_

- Be sure that you have all 18 pages of the test.
- The exam lasts for 2 hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                         |
|-------------------------|-------------------------|
| 1. (●) (b) (c) (d) (e)  | 17. (a) (b) (c) (●) (e) |
| 2. (a) (b) (c) (d) (●)  | 18. (a) (b) (c) (d) (●) |
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| 3. (a) (●) (c) (d) (e)  | 19. (a) (●) (c) (d) (e) |
| 4. (a) (●) (c) (d) (e)  | 20. (●) (b) (c) (d) (e) |
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| 5. (a) (b) (c) (●) (e)  | 21. (●) (b) (c) (d) (e) |
| 6. (●) (b) (c) (d) (e)  | 22. (a) (b) (c) (●) (e) |
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| 7. (a) (b) (●) (d) (e)  | 23. (a) (b) (●) (d) (e) |
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| 9. (a) (●) (c) (d) (e)  | 25. (a) (●) (c) (d) (e) |
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| 11. (a) (b) (c) (●) (e) | 27. (a) (b) (c) (d) (●) |
| 12. (a) (b) (●) (d) (e) | 28. (a) (b) (●) (d) (e) |
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| 13. (a) (●) (c) (d) (e) | 29. (a) (b) (c) (●) (e) |
| 14. (●) (b) (c) (d) (e) | 30. (●) (b) (c) (d) (e) |
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| 15. (a) (b) (●) (d) (e) |                         |
| 16. (a) (b) (c) (d) (●) |                         |