Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2019

Name: Answers

Instructor: Juan Migliore

Exam 1

February 7, 2019.

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

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Place an × through your answer to each problem.

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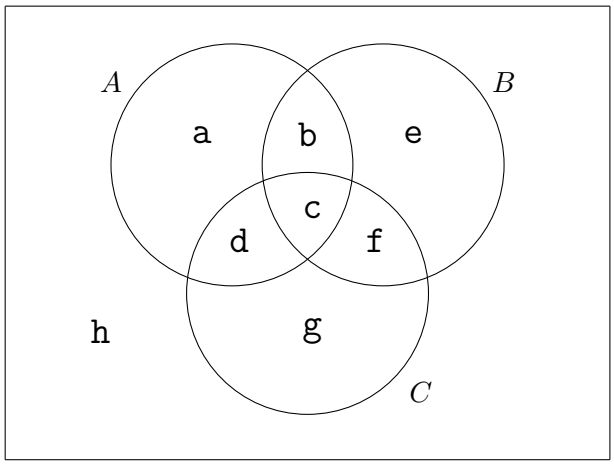
Tot. _____

Multiple Choice

1. (5 pts.) In the following Venn diagram, which of the following is equal to $(A \cap B) \cup C'$? (Note the "prime" over the C.)



either in $A \cap B$ or outside C

$= \{b, c\} \cup \{a, b, e, h\}$

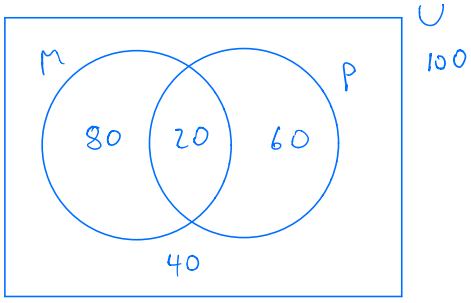
$= \{a, b, c, e, h\}$

(a) $\{a, b, e, h\}$ (b) $\{b\}$ (c) $\{a, b, c, e, h\}$

(d) $\{a, b, e\}$ (e) $\{a, b, c, e, g\}$

2. (5 pts.) A small technical college has 200 students. They have 100 math majors, 80 physics majors, and 20 double majors in both math and physics. How many students are majoring in physics but not math? (A Venn diagram might be helpful.)

(a) 60 (b) 40 (c) 80 (d) 100 (e) none



3. (5 pts.) Bob owns four pairs of pants and six shirts. Before a trip, he wants to choose two pairs of pants and three shirts to pack in his suitcase. In how many ways can he do this?

(a) $C(4,2) + C(6,3)$ (b) $P(4,2) + P(6,3)$ (c) $P(4,2) \cdot P(6,3)$

(d) $4! \cdot 6!$ (e) $C(4,2) \cdot C(6,3)$

There are $C(4,2)$ choices for pants. For each of them there are $C(6,3)$ choices for shirts.

4. (5 pts.) A standard deck consists of 52 cards, with an A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K of each of four suits (clubs, diamonds, hearts and spades). So there are four A's, four 2's, four 3's, etc. In how many ways can a person be given two A's together with three other cards that are not A's (for a total of five cards)? [Hint: all that matters is which cards the person winds up with, not the order in which she receives them.]

(a) $C(4,2) + C(48,3)$ (b) $P(4,2) \cdot P(48,3)$ (c) $P(4,2) + P(48,3)$

(d) $C(4,2) \cdot C(48,3)$ (e) $4^2 \cdot 48^3$

$C(4,2) \cdot C(48,3)$

1 9

choose the two A's

choose the three other cards

5. (5 pts.) A license plate in a certain state features 3 letters (repetition NOT allowed) followed by 3 digits (repetition IS allowed). How many different license plates are possible? [Hint: there are 26 letters and 10 digits. Obviously ABC-112 is different from CBA-211.]

(a) $P(26,3) \cdot 10^3$ (b) $P(26,3) \cdot P(10,3)$ (c) $C(26,3) \cdot C(10,3)$

(d) $C(26,3) \cdot 10^3$ (e) $26^3 \cdot 10^3$

$P(26,3) \cdot 10^3$

↑ choose the three letters ↑ choose the three digits

6. (5 pts.) Suppose I flip a coin seven times and record the sequence of heads and tails (e.g. HHTHTTT). How many such sequences are there with exactly three heads and four tails?

(a) 1225 (b) 35 (c) 12 (d) 7 (e) 128

$C(7,3) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$

↑ choose which 3 flips are H

7. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A Poker hand consists of 5 cards from this deck.

How many Poker hands have three spades and two clubs?

(a) $C(13,5)$ (b) $C(13,3)+C(13,2)$ (c) $P(13,3)\cdot P(13,2)$

(d) $C(52,3)+C(52,2)$ (e) $C(13,3)\cdot C(13,2)$

choose the spades (there are 13, you choose 3) — $C(13,3)$

" " clubs (" " " " " 2) $C(13,2)$

8. (5 pts.) A club has 9 members, and they have to choose a president, a vice president and a treasurer. In how many ways can they do this, assuming that no one is allowed to hold two of the offices?

(a) 24 (b) 504 (c) 84 (d) 729 (e) 27

$P(9,3) = 9\cdot 8\cdot 7 = 504$

9. (5 pts.) Suppose I roll a normal 6-sided die seven times and record the resulting sequence of numbers. How many sequences are there that contain **exactly** three 4's? (Don't forget that there are **seven** rolls, and that there are five other numbers that can come up besides a 4, and numbers may be repeated.)

(a) $C(7,3)$ (b) $P(7,3)$ (c) $C(7,3) \cdot 5^4$

(d) 6^3 (e) $C(7,3) \cdot 6^4$

$C(7,3) \cdot 5^4$

↑ decide the other rolls that are not 4's

↑ choose which rolls are 4's

10. (5 pts.) A club has 10 members. They are planning a big party, and they need to choose an organizing committee. This committee has to have **at least** two members (it could be more than two, but it can't consist of no one, and it can't consist of just one member). In how many ways can they choose this committee?

(a) $2^{10} = 1024$ (b) $2^{10} - 10 = 1014$ (c) $C(10,2) - 1 - 10 = 34$

(d) $2^{10} - 1 - 10 = 1013$ (e) $C(10,2) = 45$

$2^{10} - 10 - 1$

↑ all possible subsets

↑ subsets with 1 element

↑ empty set

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) A club has 9 members: 4 men and 5 women. For each of the following parts of this problem, give a numerical answer (e.g. if the answer should be $C(4,2)$, write 6.) These questions should be assumed to be independent of each other.

(a) In how many ways can they choose an executive committee of four people, if gender is irrelevant?

$C(9,4) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2} = 9 \cdot 7 \cdot 2 = 126$

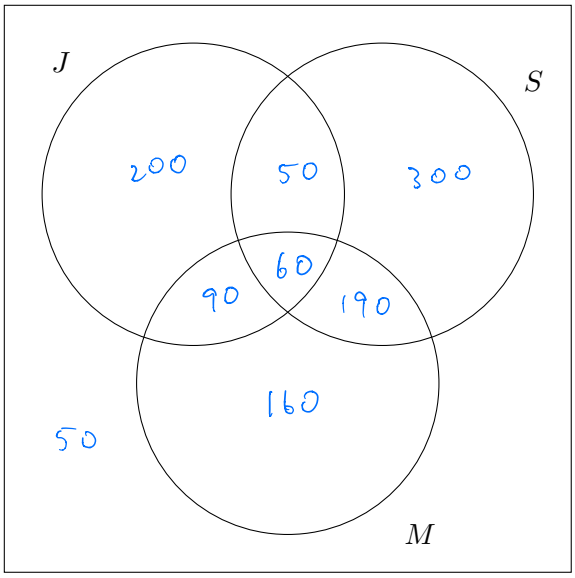
(b) In how many ways can they choose an executive committee consisting of 2 men and 2 women?

$C(4,2) \cdot C(5,2) = 6 \cdot 10 = 60$

12. (10 pts.) A certain college has 1100 students. We have the following information about the clubs that they belong to.

- 400 belong to the juggling club.
- 600 belong to the science club.
- 500 belong to the Mock Trial club
- 110 belong to both the juggling club and the science club.
- 250 belong to both the science club and the Mock Trial club.
- 150 belong to both the juggling club and the Mock Trial club.
- 60 belong to all three clubs.

Fill in **all** regions of the following Venn diagram, where J represents the juggling club, S represents the science club and M represents the Mock Trial club.



13. (10 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 10 times and write down the sequence of heads and tails that shows up.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) How many different sequences of heads and tails are possible?

2^{10}

(b) In how many ways can I get a total of 5 heads with the first and last toss being heads?

H _ _ _ _ _ _ _ _ H

you want three of these 8 to be H

$C(8,3)$

14. (10 pts.)

A bag contains 9 colored marbles, of which 5 are red and 4 are blue. (Assume that the marbles are distinguishable from each other.) I plan to pick 3 marbles from the bag.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) What is the total number of ways 3 marbles can be selected (ignoring color)?

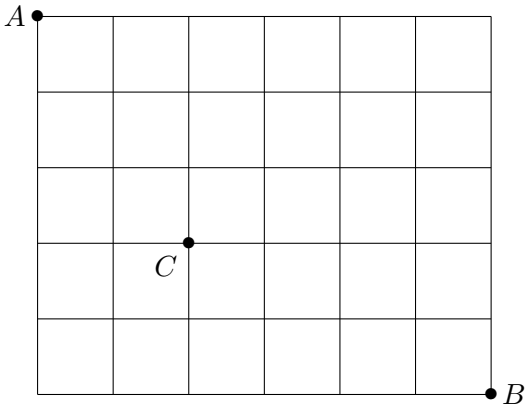
$C(9,3)$

(b) If I pick 3 marbles, in how many ways can I get all red marbles or all blue marbles?

$C(5,3) + C(4,3)$

15. (10 pts.) In this problem, be sure to show all your work and be sure to plainly mark your answers.

The following is a street map of part of a city. Emily lives at the northwest corner (marked A) and wants to get to the library at the southeast corner (marked B). She only travels east and south (i.e. to the right or down), following the roads. She happens to owe \$50 to Claire, who lives at the corner marked C.



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(a) If she decides to be conscientious and pay Claire her \$50, how many different routes are there from A to B that DO pass through C?

There are 5 blocks from A to C and 6 blocks from C to B

$C(5,3) \cdot C(6,3)$ or $C(5,2) \cdot C(6,3)$

(b) If she decides she wants to hang on to her money for now, how many different routes are there from A to B that do NOT pass through C?

$C(11,5) - C(5,3) \cdot C(6,3)$

↙ total # of routes from A to B

↘ ans. to (a)

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