Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
[redacted]
Spring 2019

Name: Solutions

Instructors: [redacted] Migliore

[redacted] Practice exam 3C

[redacted]

This exam is in two parts on 13 pages and contains 13 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

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Place an $\times$ through your answer to each problem.

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MC.	________
11.	________
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Tot.	________

Multiple Choice

1. (5 pts.) The recovery rate for a certain fish disease is 25%. An aquarium has 8 fish with the disease. What is the probability that exactly two of them recover (to the nearest two decimal places)?

(a) 0.50 (b) 0.31 (c) 0.25 (d) 1.00 (e) 0.13

Answer: Since $25\% = \frac{1}{4}$, we have $p = \frac{1}{4}$ and $q = 1 - p = \frac{3}{4}$. So the desired probability is

$$\binom{8}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6 = 0.31$$

(to the nearest two decimal places).

2. (5 pts.) In the following probability distribution

outcome	probability
12	1/2
15	1/3
18	1/6

the mean is 14 (you don't have to verify this). Find the variance.

(a) 2 (b) 3 (c) 1 (d) 4 (e) 5

Answer:

$$\frac{1}{2}(12-14)^2 + \frac{1}{3}(15-14)^2 + \frac{1}{6}(18-14)^2 = 2 + \frac{1}{3} + \frac{16}{6} = 5.$$

3. (5 pts.) Let Z be a random variable corresponding to the standard normal curve. Find $Pr(-1 \leq Z \leq 2)$. Use the table at the end of the exam.

(a) 0.7623 (b) 0.8185 (c) 0.1359

(d) .9987 (e) 0.9772

Answer:

$$A(2) - A(-1) = .9772 - .1587 = .8185.$$

4. (5 pts.) Let Z be a random variable corresponding to the standard normal curve. Find the value of b so that $Pr(0.5 \leq Z \leq b) = 0.2590$. Use the table at the end of this exam.

(a) $b = 1.65$ (b) $b = 0.65$ (c) $b = 1.55$

(d) $b = 0.5325$ (e) $b = 0.75$

Answer:

$$A(b) - A(0.5) = .2590 \implies A(b) - .6915 = .2590 \implies A(b) = .6915 + .2590 = .9505 \implies b = 1.65.$$

5. (5 pts.) Assume that the heights of NBA players are normally distributed, with a mean of 80 inches, and a standard deviation of 4 inches. If an NBA player is chosen at random, what is the probability that he is at least 85 inches tall? Use the table at the end of this exam.

(a) 8.72% (b) 9.28% (c) 89.44% (d) 10.56% (e) 0%

Answer:

Let X be the random variable associated to this problem. Then

$$Pr(X \geq 85) = 1 - Pr(X \leq 85).$$

Now,

$$z = \frac{85 - 80}{4} = \frac{5}{4} = 1.25$$

so

$$Pr(X \geq 85) = 1 - A(1.25) = 1 - .8944 = .1056 = 10.56\%.$$

6. (5 pts.) Let X be a random variable associated to a normal curve with mean $\mu = 25$ and standard deviation σ. If $Pr(X \leq 30) = 97.72\%$, find the value of σ.

(a) $\sigma = 1.5$ (b) $\sigma = 3$ (c) $\sigma = 1$ (d) $\sigma = 2.5$ (e) $\sigma = 2$

Answer:

97.72% = .9772 is $A(2.00)$, so

$$2.00 = \frac{x - \mu}{\sigma} = \frac{30 - 25}{\sigma} = \frac{5}{\sigma}.$$

Thus

$$2\sigma = 5,$$

so

$$\sigma = \frac{5}{2} = 2.5.$$

7. (5 pts.) Find the intersection point of the lines $2x + 3y = 13$ and $x + 2y = 8$. (Careful: don't do this too quickly!)

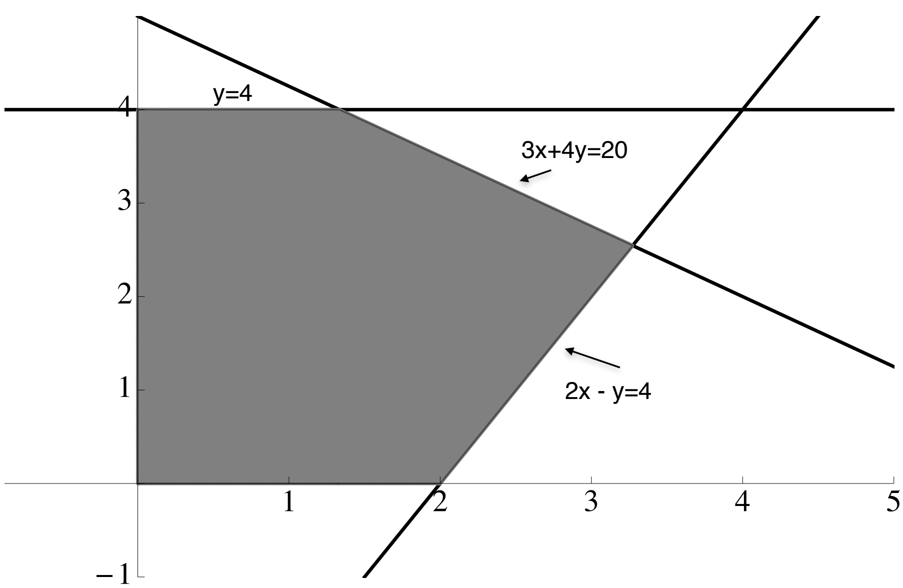
(a) $(2, 3)$ (b) $(4, 2)$ (c) $(3, 2)$ (d) $(1, 2)$ (e) $(5, 1)$

Answer:
You can solve the system

$$\begin{cases} 2x & + & 3y & = & 13 \\ x & + & 2y & = & 8 \end{cases}$$

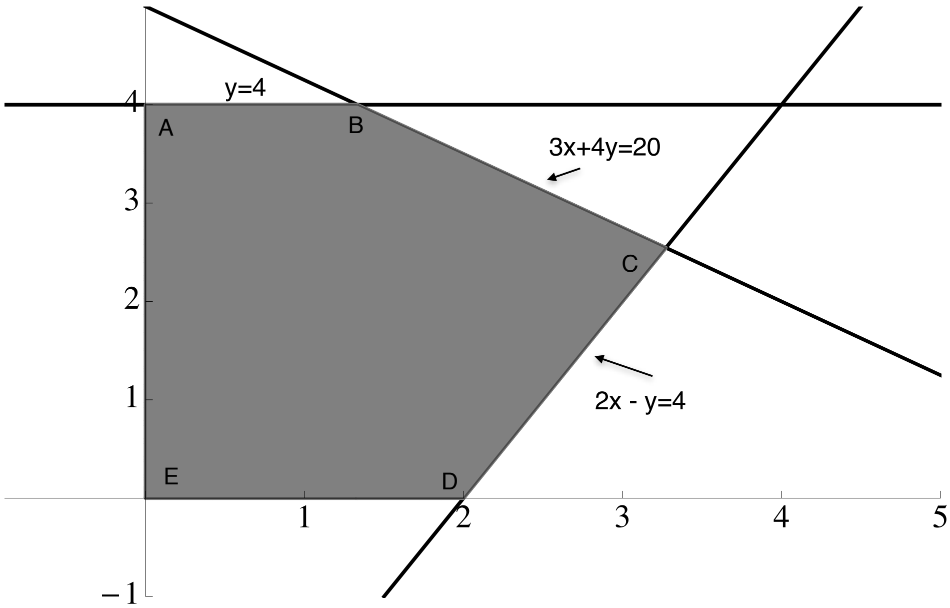
or you can plug in the answers and see which works. But be careful to check both equations, not just one!!! The answer is $x = 2,\ y = 3$, i.e. (e).

8. (5 pts.) What is the maximum of the objective function (rounded off to two decimal places) $2x+4y$ on the feasible set shown as the shaded region in the diagram below?



(a) 22.48 (b) 13.33 (c) 18.67 (d) 18.18 (e) 16.73

Answer:



We label the vertices A, B, C, D and E as shown.

The co-ordinates of E are $(0, 0)$, the co-ordinates of A are $(0, 4)$ and the co-ordinates of D are $(2, 0)$.

At B, $y = 4$ and $3x + 4y = 20$ which gives $3x + 16 = 20$ and $x = 4/3$. Thus the co-ordinates of B are $(4/3, 4)$.

At C, $2x-y = 4$ gives $y = 2x-4$. Plugging this into the equation $3x+4y = 20$ gives $3x+4(2x-4) = 20$ which gives $11x-16 = 20$ or $11x = 36$. Thus $x = 36/11$. Plugging this into $y = 2x-4$ gives $y = 28/11$. Therefore the co-ordinates of C are $(36/11, 28/11)$.

We check the value of the objective function on each vertex:

vertex	$2x + 4y$
$(0, 0)$	0
$(0, 4)$	16
$(2, 0)$	4
$(4/3, 4)$	18.67
$(36/11, 28/11)$	16.73

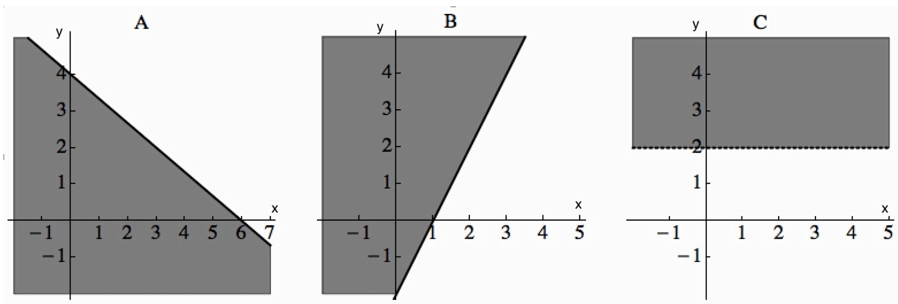
The maximum is 18.67.

9. (5 pts.)

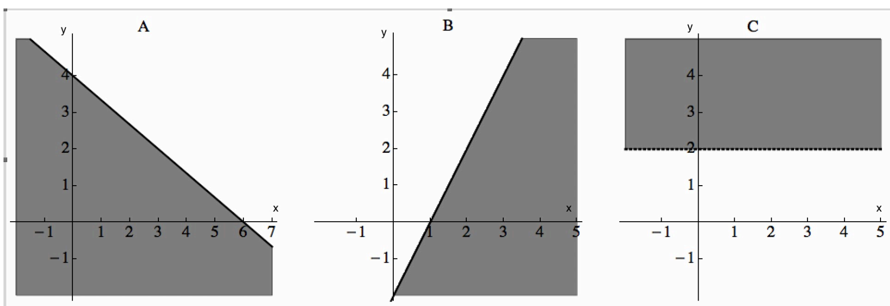
Below we give three inequalities A, B and C:

$$\begin{array}{lccccc} \text{A:} & 2x & + & 3y & \geq & 12 \\ \text{B:} & 2x & - & y & \leq & 2 \\ \text{C:} & & & y & > & 2 \end{array}$$

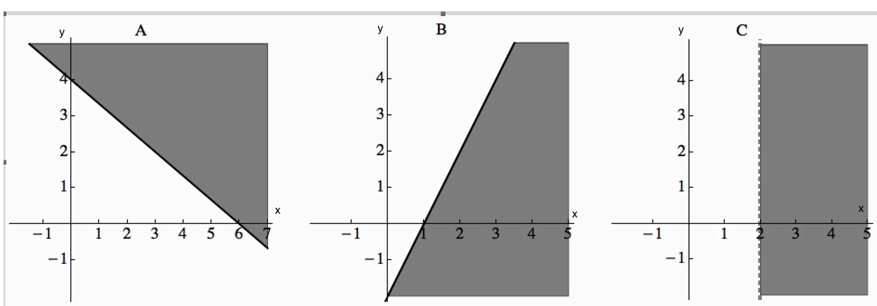
Which of the pictures below shows the correct graphs of A, B and C?



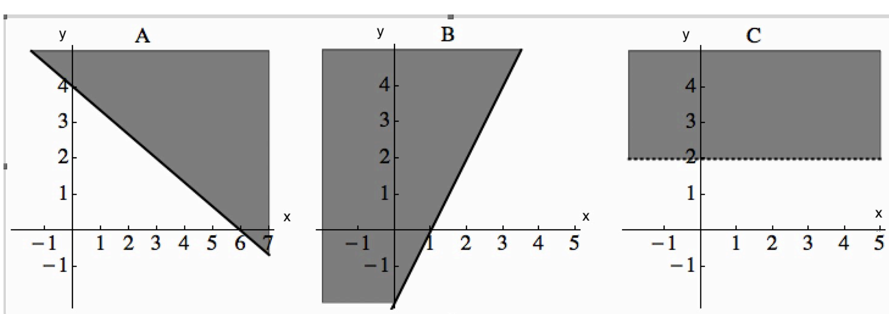
(a)



(b)



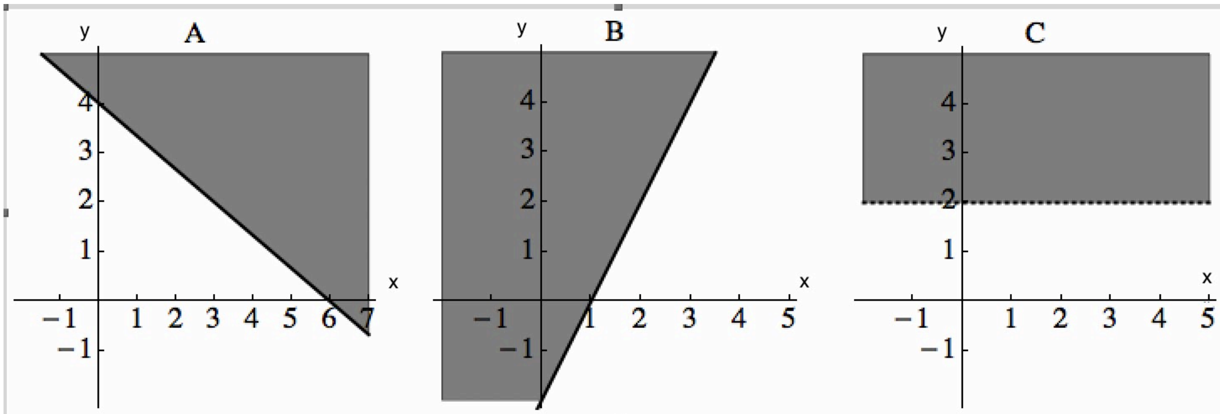
(c)



(d)

(e) None of the above

Answer: For A the point $(0,0)$ is not in the solution set (and not on the corresponding line), so the solution set is above the line $2x+3y=12$. The graph includes the line.
For B the point $(0,0)$ is in the solution set (and not on the corresponding line), so the solution set is above the line $2x-y=2$. The graph includes the line.
The graph of the inequality C is the region above the horizontal line $y=2$, the graph does not include the line. Thus the solution is



10. (5 pts.) Joe and Mary run a small business producing tables and chairs. Joe produces the parts for the furniture and Mary assembles the furniture. It takes Joe 10 hours to produce the parts for a table and it takes him 4 hours to produce the parts for one chair. It takes Mary one hour to assemble a table and two hours to assemble a chair. Joe has 40 hours to devote to making furniture parts each week and Mary has 8 hours to devote to assembling furniture each week. Each table sold brings a profit of \$200 and each chair sold brings a profit of \$50. Mary and Joe sell all of the furniture that they produce and wish to maximize profits. Let x denote the number of tables that Mary and Joe make in a week and let y denote the number of chairs they make in a week, which of the following give the constraints on x and y and the objective function?

(a)

	4x +	10y	≤	40
	2x +	y	≤	8
		y	≥	0
		x	≥	0
Obj.	Function:	50x	+	200y

(b)

	10x +	2y	≤	8
	x +	4y	≤	40
		y	≥	0
		x	≥	0
Obj.	Function:	200x	+	50y

(c)

	x +	4y	≤	40
	10x +	2y	≤	8
		y	≥	0
		x	≥	0
Obj.	Function:	50x	+	200y

(d)

	10x +	y	≤	40
	4x +	2y	≤	8
		y	≥	0
		x	≥	0
Obj.	Function:	200x	+	50y

(e)

	10x +	4y	≤	40
	x +	2y	≤	8
		y	≥	0
		x	≥	0
Obj.	Function:	200x	+	50y

Answer:

Joe's time constraint translates to $10x + 4y \leq 40$
Mary's time constraint translates to $x + 2y \leq 8$.
In addition, we must have $x \geq 0$ and $y \geq 0$. Profit is given by $200x + 50y$.

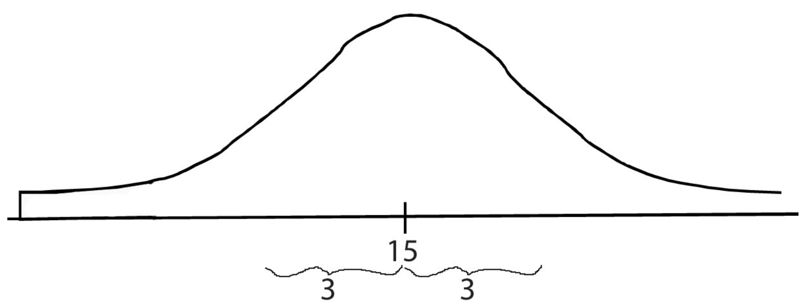
Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (15 pts.) The Wooly Sweater Company ships sweaters in bins containing 1000 sweaters each. The number of sweaters in a bin that are ruined by insects is normally distributed, with a mean of 15 and a standard deviation of 3.

(a) Sketch a normal curve for this random variable, indicating where the 15 and the 3 can be found in the curve.

Answer:



(b) The Sweaters-R-Us company of Mytown, PA, receives a bin from the Wooly Sweater Company. What is the probability that it contains at most 21 ruined sweaters? Use the table at the end of this exam.

Answer:
Let X be the random variable counting the number of ruined sweaters in a bin. We want $Pr(X \leq 21)$. The z-score corresponding to $x = 21$ is

$$\frac{21 - 15}{3} = 2.$$

Then

$$Pr(X \leq 21) = Pr(Z \leq 2) = .9772 = 97.72\%.$$

(c) These sweaters are selling like hotcakes, and the Sweaters-R-Us company is taking pre-orders for the next bin that will arrive. They want to be at least 95% sure that there will be enough unruined sweaters to cover the pre-orders. How many pre-orders should they accept? (Hint: they are 95% sure that the number of *bad* sweaters will be at most how many?)

Answer:

Again let X be the random variable counting the number of ruined sweaters in a bin. Let Z be the random variable corresponding to the standard normal curve. The table at the back of the exam gives that $A(1.65) = .9505$. Note that

$$1.65 = \frac{x - 15}{3} \implies x = 19.95.$$

This means $Pr(X \leq 20) \approx .95$. So there is a 95% probability that the number of bad sweaters will be ≤ 20. So they should accept $1000 - 20 = 980$ pre-orders.

12. (15 pts.) Every day the state lottery randomly chooses a whole number from 1 to 100 (inclusive), all equally likely to occur.

(a) Every day Bob chooses a number. What is his probability of getting it right on any given day? What is his probability of getting it wrong?

Answer:
Probability of getting it right is $p = \frac{1}{100}$. Probability of getting it wrong is $q = \frac{99}{100}$.

(b) If he plays this game 400 times, what is the expected value for the number of correct guesses?

Answer:
$n = 400$ means $\mu = np = 400 \cdot \frac{1}{100} = 4$.

(c) If he plays this game 99 times, what is the variance?

Answer:
$\sigma^2 = npq = 99 \cdot \frac{1}{100} \cdot \frac{99}{100} = \frac{9801}{10000}$.

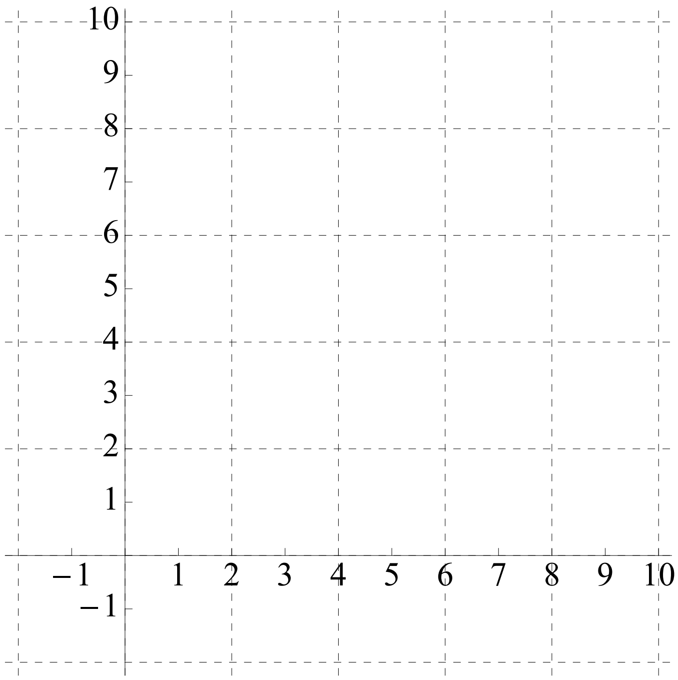
(d) If he plays this game 99 times, what is the standard deviation?

Answer:
$\sigma = \sqrt{\sigma^2} = \frac{99}{100}$.

13. (15 pts.)
Consider the set of inequalities given by:

$$\begin{array}{rrcl} 2y+ & x & \leq & 8 \\ y+ & 3x & \geq & 9 \\ & x & \geq & 0 \\ & y & \geq & 0 \\ & x & \leq & 6 \end{array}$$

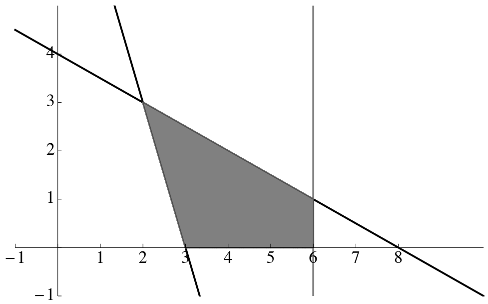
(a) Graph the feasible set corresponding to this set of inequalities on the set of axes provided. (Make sure you shade the region corresponding to the feasible set and clearly identify the region as the feasible set.)



(b) Find the vertices of the above feasible set.

(c) Find the **minimum** value of the objective function $5x + 4y$ on the above feasible set.

Answer:



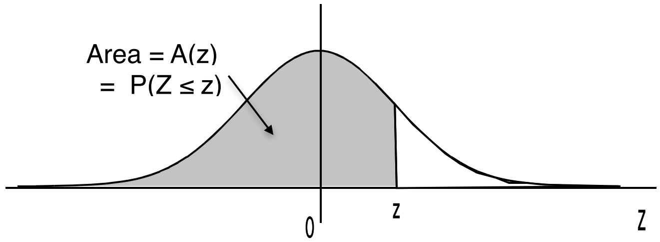
The co-ordinates of the vertex D are $(3, 0)$ and
the co-ordinates of the vertex C are $(6, 0)$.
At B, we have $x = 6$ and $2y + x = 8$, thus $2y = 2$ and $y = 1$. Therefore the co-ordinates at B are $(6, 1)$.
At A, we have $y = 9 - 3x$. Substituting this into $2y + x = 8$, we get $2(9 - 3x) + x = 8$ or $18 - 5x = 8$, giving us that $x = 2$. This gives us that $y = 9 - 3(2) = 3$. Thus the co-ordinates of A are $(2, 3)$.

We check the value of the objective function on each vertex:

vertex	$5x + 4y$
$(3, 0)$	15
$(6, 0)$	30
$(6, 1)$	34
$(2, 3)$	22

The minimum is 15.

Areas under the Standard Normal Curve



z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228	−.50	.3085	1.00	.8413	2.50	.9938
−3.45	.0003	−1.95	.0256	−.45	.3264	1.05	.8531	2.55	.9946
−3.40	.0003	−1.90	.0287	−.40	.3446	1.10	.8643	2.60	.9953
−3.35	.0004	−1.85	.0322	−.35	.3632	1.15	.8749	2.65	.9960
−3.30	.0005	−1.80	.0359	−.30	.3821	1.20	.8849	2.70	.9965
−3.25	.0006	−1.75	.0401	−.25	.4013	1.25	.8944	2.75	.9970
−3.20	.0007	−1.70	.0446	−.20	.4207	1.30	.9032	2.80	.9974
−3.15	.0008	−1.65	.0495	−.15	.4404	1.35	.9115	2.85	.9978
−3.10	.0010	−1.60	.0548	−.10	.4602	1.40	.9192	2.90	.9981
−3.05	.0011	−1.55	.0606	−.05	.4801	1.45	.9265	2.95	.9984
−3.00	.0013	−1.50	.0668	.00	.5000	1.50	.9332	3.00	.9987
−2.95	.0016	−1.45	.0735	.05	.5199	1.55	.9394	3.05	.9989
−2.90	.0019	−1.40	.0808	.10	.5398	1.60	.9452	3.10	.9990
−2.85	.0022	−1.35	.0885	.15	.5596	1.65	.9505	3.15	.9992
−2.80	.0026	−1.30	.0968	.20	.5793	1.70	.9554	3.20	.9993
−2.75	.0030	−1.25	.1056	.25	.5987	1.75	.9599	3.25	.9994
−2.70	.0035	−1.20	.1151	.30	.6179	1.80	.9641	3.30	.9995
−2.65	.0040	−1.15	.1251	.35	.6368	1.85	.9678	3.35	.9996
−2.60	.0047	−1.10	.1357	.40	.6554	1.90	.9713	3.40	.9997
−2.55	.0054	−1.05	.1469	.45	.6736	1.95	.9744	3.45	.9997
−2.50	.0062	−1.00	.1587	.50	.6915	2.00	.9772	3.50	.9998
−2.45	.0071	−.95	.1711	.55	.7088	2.05	.9798		
−2.40	.0082	−.90	.1841	.60	.7257	2.10	.9821		
−2.35	.0094	−.85	.1977	.65	.7422	2.15	.9842		
−2.30	.0107	−.80	.2119	.70	.7580	2.20	.9861		
−2.25	.0122	−.75	.2266	.75	.7734	2.25	.9878		
−2.20	.0139	−.70	.2420	.80	.7881	2.30	.9893		
−2.15	.0158	−.65	.2578	.85	.8023	2.35	.9906		
−2.10	.0179	−.60	.2743	.90	.8159	2.40	.9918		
−2.05	.0202	−.55	.2912	.95	.8289	2.45	.9929		

Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Fall 2017

Name:________________________________

Instructors: Basit & Migliore

Practice Exam III #2 - Answers

November 14, 2017

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7.	(a)	(b)	(c)	(d)	(e)
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