Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2020

Name: Solutions

Instructor: Juan Migliore

Exam 1

February 6, 2020.

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

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Place an × through your answer to each problem.

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3.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(b)	(c)	(•)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(•)	(c)	(d)	(e)
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8.	(a)	(•)	(c)	(d)	(e)
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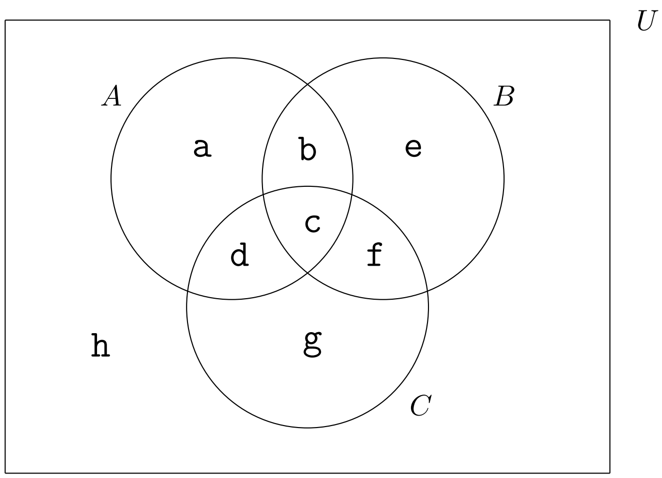
10.	(a)	(b)	(c)	(•)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) In the following Venn diagram, which of the following is equal to $A \cap (B \cup C)'$? (Note the "prime" over the C.)



"in A and not in either B or C"

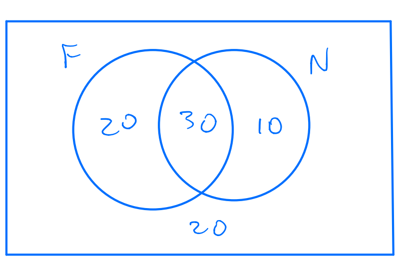
$= \{a\}$

(a) $\{a, b, c, d\}$ (b) $\{b, c, d\}$ (c) $\{a\}$

(d) $\{a, h\}$ (e) $\{e, f, g\}$

2. (5 pts.) Emily has a nice collection of poetry anthology books. Of these, 50 include poems by Robert Frost (among other authors), 40 include poems by Pablo Neruda (among other authors), 30 include poems by both Frost and Neruda (among other authors), and 20 don't have poems by either Frost or Neruda. How many poetry books are in her collection?

(a) 80 (b) 110 (c) 60 (d) 120 (e) 70



F = Frost
N = Neruda

1st put in the 30, then the 20

3. (5 pts.) Claire is running for president, and for her next campaigning trip she plans to visit four states. For her first two stops, her choices are Alabama, Alaska, Arizona and Arkansas (there are four of them). For her third and fourth stops her choices are Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri and Montana (there are eight of them). How many different 4-state campaigning trips are possible? (Order is important in this problem – Alaska then Alabama is different from Alabama then Alaska.)

(a) $C(4,2) \cdot C(8,2)$

(b) $P(4,1) + P(4,1) + P(8,1) + P(8,1)$

(c) $[P(4,1) \cdot P(4,1)] + [P(8,1) \cdot P(8,1)]$

(d) $4! \cdot 8!$

(e) $P(4,2) \cdot P(8,2)$

For her first two stops there are 4 "A" states and she has to choose two, where order matters. So she has $P(4,2)$ options. For each of those she has $P(8,2)$ options for her 3^{rd} and 4^{th} stops.

4. (5 pts.) A standard deck consists of 52 cards, with an A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K of each of four suits (clubs, diamonds, hearts and spades). So there are four A's, four 2's, four 3's, etc. In how many ways can a person be given a 2, a 3, a 4, a 5 and a 6?

(a) $\dfrac{C(4,1)}{5!}$

(b) $P(4,1)^5$

(c) $\dfrac{P(4,1)}{5!}$

(d) $C(4,1)^5$

(e) $C(4,1) + C(4,1) + C(4,1) + C(4,1) + C(4,1)$

There are $C(4,1)$ ways to choose an A. For each of these there are $C(4,1)$ ways to choose a 2. And so on. So we have

$$C(4,1) \cdot C(4,1) \cdot C(4,1) \cdot C(4,1) \cdot C(4,1) = C(4,1)^5$$

5. (5 pts.) When you get an account at a certain website, you have to choose a password consisting of five letters (with NO repetition) followed by two digits (repetitions allowed). How many different passwords are possible? [Hint: there are 26 letters and 10 digits. Obviously ABCDE-33 is different from CEBDA-33.]

(a) ~~X~~ $P(26,5) \cdot 10^2$ (b) $P(26,5) \cdot P(10,2)$ (c) $C(26,5) \cdot C(10,2)$

(d) $C(26,5) \cdot 10^2$ (e) $26^5 \cdot 10^2$

$\underline{26}\ \underline{25}\ \underline{24}\ \underline{23}\ \underline{22} \cdot \underline{10}\ \underline{10}$

$P(26,5) \cdot 10^2$

6. (5 pts.) A die has six sides, labelled 1 to 6. Suppose I roll the die three times and record the sequence of results (e.g. 4-1-5). How many such sequences don't have **any** 3's or 4's?

(a) 200 (b) ~~X~~ 64 (c) 152 (d) 204 (e) 120

If there are no 3's or 4's, each roll has to be 1, 2, 5 or 6,

So the number of sequences is

$\underline{4}\ \underline{4}\ \underline{4} = 64$

7. (5 pts.) A club has 9 members, and they have to choose an organizing committee of three people for an upcoming event. In how many ways can they do this?

(a) 504 (b) 24 (c) 729 (d) 27 ~~(e)~~ 84

$C(9,3) = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} = 84$

8. (5 pts.) Six couples decide to go to the movies and sit together in the front row. In how many ways can these 12 people seat themselves if couples must sit together? [Hint: looking left to right, Bob-Sally is different from Sally-Bob.]

(a) 720 ~~(b)~~ 46,080 (c) 10,800

(d) 21,600 (e) 8,640

First decide the order of the couples. There are $6!$ ways to do this. then for each couple, decide who's on the left. There are 2 ways to do this for each couple. So

$6! \, 2^6 = 46{,}080$

9. (5 pts.) A dinner at Luigi's pizza consists of a pizza and a salad. They offer six different toppings, of which a customer can choose as many as they like as long as they choose at least one. They offer three different salads, of which the customer has to choose exactly one. How many dinners are possible?

(a) 192 (b) 66 ~~(c)~~ 189 (d) 67 (e) 18

pizzas: $2^6 - 1 = 63$

salads: 3

answer: $63 \cdot 3 = 189$

10. (5 pts.) A box of candy contains 30 different chocolates. Mom has three indistinguishable paper bags and decides to put 10 candies in each bag. The order of the bags makes no difference because they're indistinguishable. In how many ways can she divide the chocolates into the three bags?

(a) $\dfrac{30!}{10! \cdot 10! \cdot 10!}$ (b) $\dfrac{1}{6} \cdot \dfrac{30!}{10! \cdot 20!}$ (c) $\dfrac{30!}{10! \cdot 20!}$

~~(d)~~ $\dfrac{1}{6} \cdot \dfrac{30!}{10! \cdot 10! \cdot 10!}$ (e) $\dfrac{1}{720} \cdot \dfrac{30!}{10! \cdot 10! \cdot 10!}$

unordered partition. Note $\frac{1}{3!} = \frac{1}{6}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit!
Make sure that your answer is in the answer box. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) A club has 10 members: 4 men and 6 women. For each of the following parts of this problem, give a numerical answer (e.g. if the answer should be $C(4,2)$, write 6.) These questions should be assumed to be independent of each other.

(a) In how many ways can they choose an executive committee of four people, if gender is irrelevant?

order is not important.

$$C(10,4) = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210$$

Answer to (a):
210

← Remember this has to be a number!

(b) In how many ways can they choose an executive committee consisting of 2 men and 2 women?

$$C(4,2)\cdot C(6,2) = 6\cdot 15 = 90$$

Answer to (b):
90

← Remember this has to be a number!

12. (10 pts.) A horror movie has a cast of 250 clowns. Some have red noses (R), some have blue shirts (B) and some have green pants (G).

- 110 have red noses.
- 130 have blue shirts.
- 100 have green pants.
- 30 have both red noses and blue shirts.
- 70 have both blue shirts and green pants.
- 40 have both red noses and green pants.
- 10 have all three: red noses, blue shirts and green pants.

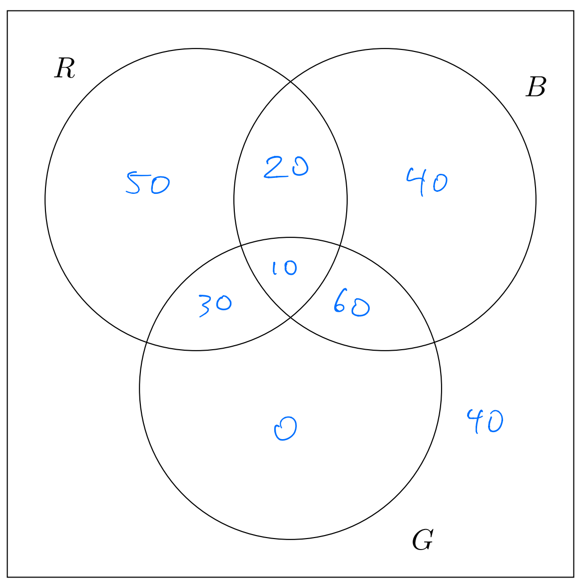
Fill in **all** regions of the following Venn diagram.

Fill in the 10 first

Then 30, 60, 20.

Then 50, 40, 0

Then 40



250

13. (10 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 12 times and write down the sequence of heads and tails that shows up.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) How many different sequences of heads and tails are possible?

12 tosses, 2 possible outcomes each time

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \ldots\ \underline{2} = 2^{12}$

Answer to (a): 2^{12}

(b) In how many ways can I get a total of 4 heads with the first being heads and the last being tails? [Hint: the first one is heads, so you just need to have three more heads.]

$\underline{H}\ \underline{\ \ }\ \ldots\ \underline{\ \ }\ \underline{T}$

10 tosses left. want 3 to be H.

Answer to (b): $\binom{10}{3} = C(10,3)$

14. (10 pts.)

A bag contains 12 colored marbles, of which 5 are red, 4 are white and 3 are blue. (Assume that the marbles are distinguishable from each other. In this problem, the order that you pick the marbles does not matter.) I plan to pick 3 marbles from the bag.

Note: In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) In how many ways can I pick the three marbles so that they are all the same color?

all red: $C(5,3)$

all white: $C(4,3)$

all blue: $C(3,3)$

Answer to (a):

$C(5,3) + C(4,3) + C(3,3)$

(b) In how many ways can I pick the three marbles so that there are strictly more red marbles than either of the other colors? [Hint: as a first step, figure out all the ways that more of the three can be red than either white or blue. A tie is no good, i.e. 1 red, 1 white and 1 blue does not contain strictly more red than white or blue.]

3 red, 0 white, 0 blue: $C(5,3) \cdot C(4,0) \cdot C(3,0) = C(5,3)$

2 red, 1 white, 0 blue: $C(5,2) \cdot C(4,1) \cdot C(3,0) = C(5,2) \cdot C(4,1)$

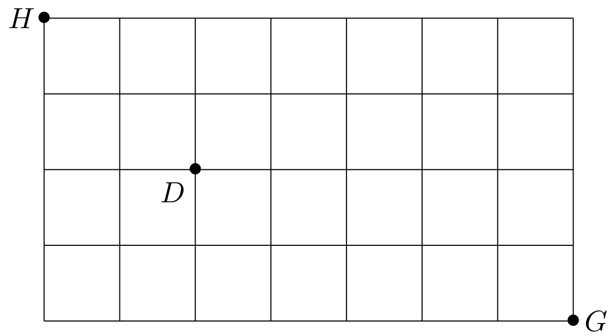
2 red, 0 white, 1 blue: $C(5,2) \cdot C(4,0) \cdot C(3,1) = C(5,2) \cdot C(3,1)$

Answer to (b):

$C(5,3) + C(5,2)C(4,1) + C(5,2)C(3,1)$

15. (10 pts.) In this problem, be sure to show all your work and be sure to use the answer box.

The following is a street map of part of the city where Harry Potter lives with his uncle and aunt. Harry lives at the northwest corner (marked H) and wants to get to the grocery store at the southeast corner (marked G). He only travels east and south (i.e. to the right or down), following the roads. Unfortunately, there is a dementor at the corner marked D.



Note: In the following two parts, you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) If he doesn't care whether he meets the dementor or not, how many paths are possible for him to get from H to G?

There are 11 blocks, of which 4 are south

Answer to (b):

$\binom{11}{4}$

(b) If he would rather not meet the dementor, how many paths are possible for him to get from H to G that do **not** pass through D?

$\binom{4}{2}$ routes from H to D, $\binom{7}{2}$ routes from D to G.

$\binom{4}{2}\cdot\binom{7}{2}$ routes that do pass through D.

Answer to (b):

$\binom{11}{4} - \binom{4}{2}\binom{7}{2}$

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MC.	________
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