Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2020

Name: Solutions

Instructor: Juan Migliore

Exam 3

April 16, 2020

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. Please copy your completed exam and return it to me by email by 4:00. You may use a calculator, books and notes, but no other aid is allowed. Be sure to write your name on this title page and sign the honor pledge.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each multiple choice problem and please sign the honor pledge.

Honor Pledge: _____

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC. _____
11. _____
12. _____
13. _____
14. _____
15. _____

Tot. _____

Multiple Choice

1. (5 pts.) A professor is teaching from home because of the pandemic, and she is trying to reconstruct the grades from the most recent (pre-pandemic) exam because she left her gradebook on campus. She remembers the following facts:

- Her class has 10 students.
- Eight of the ten scores were 78, 78, 79, 80, 81, 82, 83, 83.
- The remaining two scores were the same.
- The mean score for the exam was 82.

What score did each of the remaining two students have?

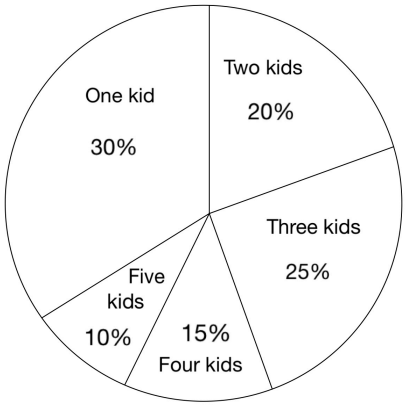
(a) 87 (b) 88 (c) 94 (d) 85 (e) 82

$$\frac{78+78+79+80+81+82+83+83+2x}{10} = 82$$

$$644 + 2x = 820$$
$$2x = 176$$
$$x = 88$$

2. (5 pts.)

The following pie chart gives the percentage of female goats that have one, two, three, four or five kids over their lifetime in the mythical country of Freedonia. What is the mean number of kids per female goat in Freedonia?



(a) 2.55 (b) 3.00 (c) 2.50 (d) 2.45

(e) It can't be determined from the given information.

$$\mu = (1)(.3) + (2)(.2) + (3)(.25) + 4(.15) + 5(.1)$$
$$= .3 + .4 + .75 + .6 + .5$$
$$= 2.55$$

3. (5 pts.) In the country of Freedonia, cats are very popular and the government wants to know how evenly the cats are spread out among the citizens. Since there are far too many people in the country for them to ask everyone, a random **sample** was chosen, with the following results. Here x_i represents the number of cats, and f_i represents the number of people in that **sample** who owned x_i cats.

x_i	f_i	$x_i f_i$	$(x_i-\mu)^2$	$f_i(x_i-\mu)^2$
8	6	48	1.21	7.26
9	8	72	0.01	0.08
10	4	40	0.81	3.24
11	2	22	3.61	7.22
	20	182		17.80

$\mu = \frac{182}{20} = 9.1$

What is the **sample variance** for this data? As a hint, the mean is $\bar{x} = 9.1$. The following answers are just to three decimal places.

(a) $s^2 = 0.297$ (b) $s^2 = 0.282$ (c) $s^2 = 1.880$

(d) $s^2 = 0.937$ (e) $s^2 = 5.9333$

Because it's a sample we divide by 19 instead of 20.

$s^2 = \frac{17.80}{19} = 0.937$

4. (5 pts.) To three decimal places, compute the standard deviation $\sigma(X)$ for the random variable defined as follows:

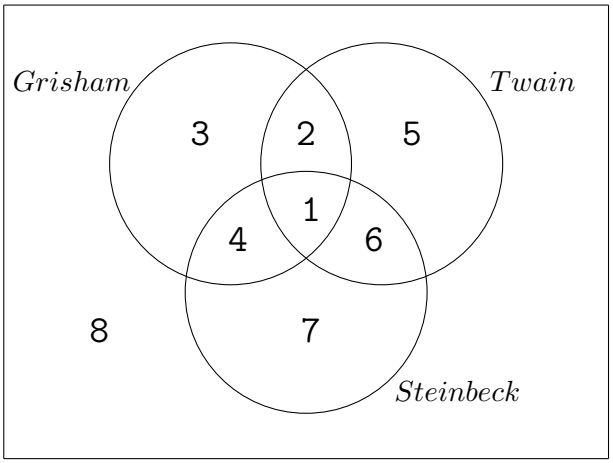
x_i	p_i	$x_i p_i$	$(x_i-\mu)^2$	$p_i(x_i-\mu)^2$
10	0.5	5	64	32.0
20	0.2	4	4	0.8
30	0.3	9	144	43.2
		18		76.0

(a) 8.406 (b) 8.718 (c) 76.000 (d) 18.000 (e) 14.560

$\sigma^2(X) = 76$

$\sigma(X) = \sqrt{76} = 8.718$

5. (5 pts.) The 36 members of the book club are queried about certain authors that they may or may not have read in the last month. The following Venn diagram shows how many have read all possible combinations of Grisham, Steinbeck and Twain. Let X be the random variable counting how many of the three authors were read by a randomly selected member of the club. Find the probability distribution for this random variable. (The probabilities are given to three decimal places.)



$P(X=0) = \frac{8}{36} = .222$

$P(X=1) = \frac{3+5+7}{36} = .417$

$P(X=2) = \frac{2+4+6}{36} = .333$

$P(X=3) = \frac{1}{36} = .028$

(a)

k	$P(X=k)$
0	0222
1	0.278
2	0.389
3	0.500

(b)

k	$P(X=k)$
1	0.278
2	0.389
3	0.500

(c)

k	$P(X=k)$
1	0.536
2	0.429
3	0.036

(d)

k	$P(X=k)$
0	0.250
1	0.250
2	0.250
3	0.250

(e)

k	$P(X=k)$
0	0.222
1	0.417
2	0.333
3	0.028

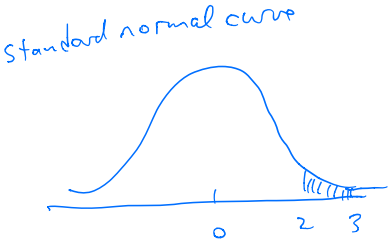
6. (5 pts.) Bizarro works at a booth in a carnival, where he guesses the age of people who stop by. He is correct in his guess 75% of the time. If 8 people come by on a given day, what is the probability that he will guess exactly 6 of their ages correctly?

(a) 5.25% (b) 75% (c) 31.1% (d) 0.4% (e) 26.8%

$C(8,6)(.75)^6(.25)^2 \approx 0.311 = 31.1\%$

7. (5 pts.) In any normal distribution, find the area between two standard deviation units and three standard deviation units to the right of the mean. [Hint: use the table at the end of this exam.]

(a) 0.0214 (b) 0.9760 (c) 0.9786 (d) 0.4987 (e) 0.4773

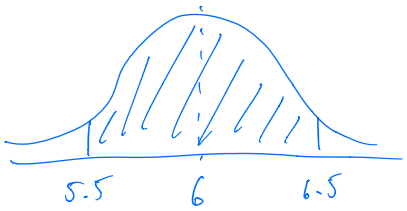


$A(3) - A(2) = 0.4987 - 0.4773$

$= .0214$

8. (5 pts.) The heights of all people in Freedonia are normally distributed, with a mean of 6 feet and a standard deviation of 0.25 feet. A person is chosen at random. What is the probability that she is between 5.5 feet and 6.5 feet?

(a) 47.73% (b) 68.26% (c) 34.13% (d) 95.46% (e) 19.74%

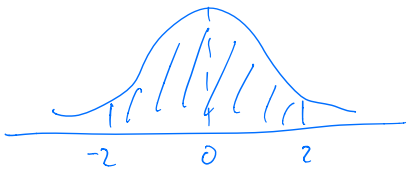


$\rightsquigarrow$

$x = 6.5 \rightsquigarrow z = \frac{6.5 - 6}{.25} = 2$

$x = 5.5 \rightsquigarrow z = \frac{5.5 - 6}{.25} = -2$

SNC



$2 \cdot A(2) = 2(0.4773)$

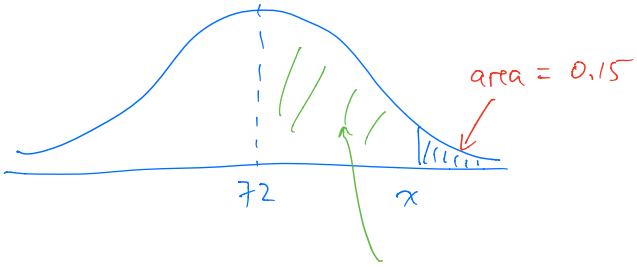
$= 0.9546$

$= 95.46\%$

9. (5 pts.) In the country of Freedonia, the weights of dogs are normally distributed with a mean of 72 pounds and a standard deviation of 5 pounds. (They keep big dogs in Freedonia to discourage tourism.) Nicole bragged that only 15% of the dogs in Freedonia are heavier than her dog. How much does Nicole's dog weigh?

(a) 73.95 pounds (b) 77.20 pounds (c) 82.80 pounds

(d) 72.68 pounds (e) 72.30 pounds



What z satisfies $A(z) = 0.35$ (find the closest one)? From the table,

$z = 1.04$

Convert to x

$$\frac{x-72}{5} = 1.04$$

$$x - 72 = 5.20$$

$$x = 77.20$$

10. (5 pts.) Multiplications-R-Us is a company that produces hand calculators. Unfortunately, 25% of the calculators they produce are defective. If Watson Elementary School bought 100 calculators from the company, use the normal approximation to the binomial distribution to estimate the probability that between 20 and 35 of the calculators they bought are defective.

(a) 9.42% (b) 86.45% (c) 11.47% (d) 3.75% (e) 89.02%

Want $P(19.5 \le X \le 35.5)$

$n = 100$

$p = .25$

$q = .75$

$\mu = np = 25$

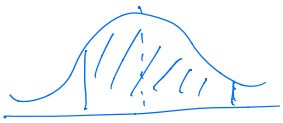
$\sigma = \sqrt{npq}$

$= \sqrt{18.75}$

$= 4.33$

$x = 19.5 \rightsquigarrow z = \frac{19.5 - 25}{4.33} = -1.27$

$x = 35.5 \rightsquigarrow z = \frac{35.5 - 25}{4.33} = 2.42$



$A(2.42) + A(1.27)$

$= .4922 + .3980$

$= .8902$

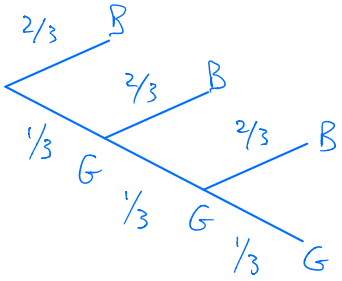
$= 89.02\%$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. **You're more likely to get partial credit for a wrong answer if you explain your reasoning**.

11. (10 pts.) An urn contains 2 blue balls and 1 green one. A game consists of repeatedly choosing a ball from the urn, recording if it is blue or green, and then returning the ball to the urn. You stop when you have recorded either **one** blue ball or **three** green balls. Let X be the random variable that counts how many balls (total) you picked in this game.

(a) Draw a tree diagram to represent this game. Be sure to label the probabilities on the branches.



you stop when you have either one blue or three green, so these are the only possibilities

(b) What are the possible values for X?

Answer to (b):

1, 2, 3

(c) Write the probability distribution for X. (Your answers to (b) should be in the first column.) Use the extra space for your calculations, and just put your answers in the table. I'd like the answers in the second column to be either fractions or decimals. Be sure to show your work and put your answer in the box.

Answer to (c):

k	$P(X = k)$
1	2/3
2	2/9
3	1/9

$P(X=1) = \frac{2}{3}$ one blue

$P(X=2) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$ green - blue

$P(X=3) = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$ GGB GGG

$= \frac{1}{9}$

12. (10 pts.) María has six teddy bears and four stuffed rabbits in her toy bin, all mixed together. She randomly selects two of these toys to take with her to bed. Let X be the random variable counting the number of teddy bears she picks.

(a) List the possible values of X.

Answer to (a):

0, 1, 2

(b) Write out the probability distribution for the random variable X. Please write the probabilities as fractions. Be sure to show your work and put your answer in the box.

Answer to (b):

k	$P(X = k)$
0	2/15
1	8/15
2	1/3

$$P(X=0) = \frac{C(4,2)}{C(10,2)} = \frac{6}{45} = \frac{2}{15}$$

$$P(X=1) = \frac{C(6,1)C(4,1)}{C(10,2)} = \frac{24}{45} = \frac{8}{15}$$

$$P(X=2) = \frac{C(6,2)}{C(10,2)} = \frac{15}{45} = \frac{1}{3}$$

(c) What is the expected value for the number of teddy bears that she will pick? (It is not necessarily an integer. Use the area to the right, if necessary, for your calculations.) Please show your work and put your answer in the box.

Answer to (c):

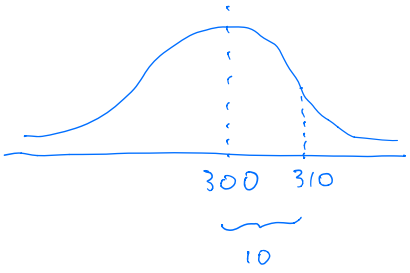
1.2

$$E(X) = (0)\left(\frac{2}{15}\right) + (1)\left(\frac{8}{15}\right) + (2)\left(\frac{1}{3}\right)$$

$$= \frac{8}{15} + \frac{10}{15} = \frac{18}{15} = \frac{6}{5} = 1.2$$

13. (10 pts.) The life span of a certain kind of insect is normally distributed, with a mean of $\mu = 300$ days and a standard deviation of $\sigma = 10$ days.

(a) Sketch this normal curve. Be sure to indicate how the mean and the standard deviation can be read off from the curve.

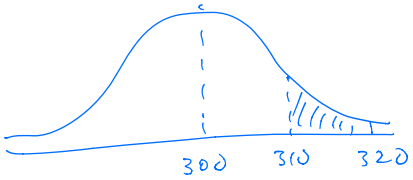


(b) What fraction of the insect population will live between 310 and 320 days? Please show your work and put your answer in the box.

Answer to (b):
13.6%

$x = 310 \rightsquigarrow z = \frac{310 - 300}{10} = 1$

$x = 320 \rightsquigarrow z = \frac{320 - 300}{10} = 2$



prob. = shaded area

$= A(2) - A(1)$

$= 0.4773 - 0.3413$

$= 0.1360$

$= 13.6\%$

14. (10 pts.) A book publisher conducts a study on people's reading preferences. They know that one quarter (1/4) of the people in a certain city prefer to read physical books while three quarters (3/4) prefer to read some sort of tablet. In the study, 100 people are asked for their preference. The publisher wants to count the number of people, in these 100, who prefer physical books ("success" for them).

(a) Find the mean, variance and standard distribution for this binomial distribution. Please show your work and put your answers in the box.

$\mu =$ 25

$\sigma^2 =$ 18.75

$\sigma =$ 4.33

$p = 1/4$, $q = 3/4$

$\mu = np = 25$

$\sigma^2 = npq = 18.75$

$\sigma = \sqrt{\sigma^2} = 4.33$

(b) What it the probability that exactly 25 of those people prefer physical books? Leave your answer in the form of binomial coefficients and powers, not an actual numerical answer.

Answer to (b):

$C(100, 25)\left(\frac{1}{4}\right)^{25}\left(\frac{3}{4}\right)^{75}$

(c) Use the normal approximation to the binomial distribution to estimate the probability that **at most** 35 of the people prefer physical books. Be sure to show your work, including a sketch of a normal curve, shading the region you're looking for. (The sketch should not go in the box, only your answer should.)

Answer to (c):

0.9922 = 99.22%

$\mu = 25$, $\sigma = 4.33$

Want $P(X \le 35.5)$

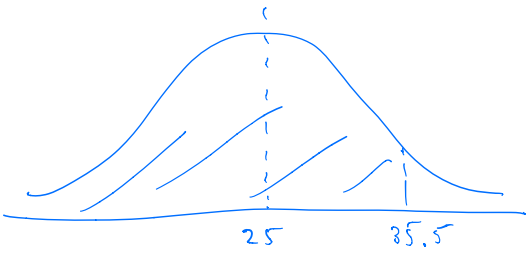
$x = 35.5 \rightsquigarrow z = \frac{35.5 - 25}{4.33} = 2.42$

Answer $= 0.5 + A(2.42)$

$= 0.5 + 0.4922$

$= 0.9922$

$= 99.22\%$



15. (10 pts.) Bruno and Hugo play the following game. They pick a card at random from a well-shuffled standard deck of 52 cards (remember there are four A's, four 1's, four 2's, ... , four 10's, four J's, four Q's and four K's).

- If the card is an A, 2, 3 or 4 then Bruno pays Hugo \$10.
- If the card is a 5, 6 or 7 then Hugo pays Bruno \$15.
- If the card is an 8, 9, 10, jack, queen or king then they flip a coin. If it's heads then Hugo gives Bruno \$20. If it's tails then Bruno gives Hugo \$25.

Let X be the random variable representing the amount of money paid *from Bruno's perspective*. (For example, if Bruno pays Hugo \$50 then we would have $X = -50$.)

(a) Write the probability distribution for this random variable. Please leave probabilities in fraction form.

Answer to (a):

k	$P(X = k)$
-10	$4/13$
15	$3/13$
20	$3/13$
-25	$3/13$

$P(X=-10) = 4/13$

$P(X=15) = 3/13$

$P(X=20) = \frac{6}{13} \cdot \frac{1}{2} = \frac{3}{13}$

$P(X=-25) = \frac{6}{13} \cdot \frac{1}{2} = \frac{3}{13}$

(b) What is the expected value for the random variable?

Answer to (b):

-0.77

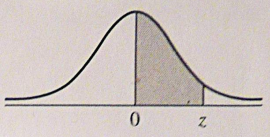
$E(X) = -10\left(\frac{4}{13}\right) + 15\left(\frac{3}{13}\right) + 20\left(\frac{3}{13}\right) + (-25)\left(\frac{3}{13}\right)$

$= \frac{-40 + 45 + 60 - 75}{13} = -0.77$

(c) Interpret your answer to (b) by filling in the blanks in the following sentence.

On average, Bruno (name)
will pay Hugo (name)
\$0.77 (amount of money)
each time the game is played. (The third blank should be a positive number.)

Area Under the Standard Normal Curve



z	A	z	A	z	A	z	A	z	A	z	A	z	A
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332	2.00	0.4773	2.50	0.4938	3.00	0.4987
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345	2.01	0.4778	2.51	0.4940	3.01	0.4987
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357	2.02	0.4783	2.52	0.4941	3.02	0.4987
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370	2.03	0.4788	2.53	0.4943	3.03	0.4988
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382	2.04	0.4793	2.54	0.4945	3.04	0.4988
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394	2.05	0.4798	2.55	0.4946	3.05	0.4989
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406	2.06	0.4803	2.56	0.4948	3.06	0.4989
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418	2.07	0.4808	2.57	0.4949	3.07	0.4989
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4430	2.08	0.4812	2.58	0.4951	3.08	0.4990
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441	2.09	0.4817	2.59	0.4952	3.09	0.4990
0.10	0.0398	0.60	0.2258	1.10	0.3643	1.60	0.4452	2.10	0.4821	2.60	0.4953	3.10	0.4990
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463	2.11	0.4826	2.61	0.4955	3.11	0.4991
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474	2.12	0.4830	2.62	0.4956	3.12	0.4991
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4485	2.13	0.4834	2.63	0.4957	3.13	0.4991
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495	2.14	0.4838	2.64	0.4959	3.14	0.4992
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505	2.15	0.4842	2.65	0.4960	3.15	0.4992
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515	2.16	0.4846	2.66	0.4961	3.16	0.4992
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525	2.17	0.4850	2.67	0.4962	3.17	0.4992
0.18	0.0714	0.68	0.2518	1.18	0.3810	1.68	0.4535	2.18	0.4854	2.68	0.4963	3.18	0.4993
0.19	0.0754	0.69	0.2549	1.19	0.3830	1.69	0.4545	2.19	0.4857	2.69	0.4964	3.19	0.4993
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554	2.20	0.4861	2.70	0.4965		
0.21	0.0832	0.71	0.2612	1.21	0.3869	1.71	0.4564	2.21	0.4865	2.71	0.4966		
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573	2.22	0.4868	2.72	0.4967		
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582	2.23	0.4871	2.73	0.4968		
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591	2.24	0.4875	2.74	0.4969		
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599	2.25	0.4878	2.75	0.4970		
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608	2.26	0.4881	2.76	0.4971		
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616	2.27	0.4884	2.77	0.4972		
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625	2.28	0.4887	2.78	0.4973		
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633	2.29	0.4890	2.79	0.4974		
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641	2.30	0.4893	2.80	0.4974		
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649	2.31	0.4896	2.81	0.4975		
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656	2.32	0.4898	2.82	0.4976		
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664	2.33	0.4901	2.83	0.4977		
0.34	0.1331	0.84	0.2996	1.34	0.4099	1.84	0.4671	2.34	0.4904	2.84	0.4977		
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678	2.35	0.4906	2.85	0.4978		
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686	2.36	0.4909	2.86	0.4979		
0.37	0.1443	0.87	0.3079	1.37	0.4147	1.87	0.4693	2.37	0.4911	2.87	0.4980		
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4700	2.38	0.4913	2.88	0.4980		
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706	2.39	0.4916	2.89	0.4981		
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713	2.40	0.4918	2.90	0.4981		
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719	2.41	0.4920	2.91	0.4982		
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726	2.42	0.4922	2.92	0.4983		
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732	2.43	0.4925	2.93	0.4983		
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738	2.44	0.4927	2.94	0.4984		
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744	2.45	0.4929	2.95	0.4984		
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750	2.46	0.4931	2.96	0.4985		
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756	2.47	0.4932	2.97	0.4985		
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4762	2.48	0.4934	2.98	0.4986		
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767	2.49	0.4936	2.99	0.4986		

Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2020

Name:_____

Instructor: Juan Migliore

Exam 3

April 16, 2020

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. Please copy your completed exam and return it to me by email by 4:00. You may use a calculator, books and notes, but no other aid is allowed. Be sure to write your name on this title page and sign the honor pledge.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each multiple choice problem and please sign the honor pledge.

Honor Pledge: _____

1.	(a)	(●)	(c)	(d)	(e)
2.	(●)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(●)	(e)
4.	(a)	(●)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(●)
6.	(a)	(b)	(●)	(d)	(e)
7.	(●)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(●)	(e)
9.	(a)	(●)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(●)

MC. _____
11. _____
12. _____
13. _____
14. _____
15. _____

Tot. _____