

## Practice Exam 2

March 5, 2020

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**You must record on this page your answers to the multiple choice problems.**

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Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (e) |
| 2.  | (a) | (b) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (e) |
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MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

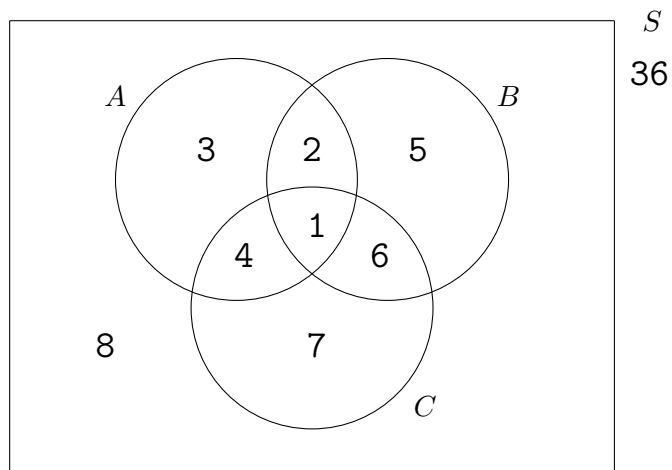
14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) The 36 members of the Travel Club have upcoming trips planned to Argentina (A), Bolivia (B) and Colombia (C). The following Venn diagram indicates how many members of the club are planning to go on the corresponding trips. If a student is chosen at random, what is the probability that she is going to Argentina?



- (a)  $\frac{3}{36}$       (b)  $\frac{10}{36}$       (c)  $\frac{9}{36}$       (d)  $\frac{3}{10}$       (e)  $\frac{1}{10}$

2. (5 pts.) Refer to the club and Venn diagram in problem #1. A student is chosen at random, and it is discovered that he is going to Argentina. With this extra information, what is the probability that he is also going to Bolivia?

- (a)  $\frac{3}{10}$       (b)  $\frac{2}{10}$       (c)  $\frac{3}{14}$       (d)  $\frac{2}{14}$       (e)  $\frac{14}{36}$

**3.** (5 pts.) Claire rolls a red die and a blue die and observes the sum. Find the probability that the sum is odd. [Note: “odd” means, in this case, either 3, 5, 7, 9 or 11.]

(a)  $\frac{12}{36} = \frac{1}{3}$

(b)  $\frac{9}{36} = \frac{1}{4}$

(c)  $\frac{5}{11}$

(d)  $\frac{6}{36} = \frac{1}{6}$

(e)  $\frac{18}{36} = \frac{1}{2}$

**4.** (5 pts.) Emily flips a coin 6 times. Find the probability that the coin shows Heads exactly 3 times. Give your answer as a fraction in lowest terms.

(a)  $\frac{1}{2}$

(b)  $\frac{1}{8}$

(c)  $\frac{5}{16}$

(d)  $\frac{1}{20}$

(e)  $\frac{3}{16}$

5. (5 pts.) In a certain town in Canada, 50% of the population speaks English, 70% speaks French and 10% doesn't speak either English or French. A person is chosen at random. What is the probability that she speaks both English and French?

- (a) 90%            (b) 10%            (c) 20%            (d) 30%            (e) 60%

6. (5 pts.) The Smith family has 10 children, consisting of four boys and six girls. After a big snowstorm, the parents randomly choose three of the children to shovel the driveway. What is the probability that **at least** one of the three is a boy?

- (a)  $\frac{5}{6}$             (b)  $\frac{1}{6}$             (c)  $\frac{1}{3}$             (d)  $\frac{1}{2}$             (e)  $\frac{2}{3}$

7. (5 pts.) In a certain city, 55% of the people are male and 45% are female. Of the males, 15% have a certain disorder. Of the females, 5% have the disorder. A person is chosen at random and found **not** to have the disorder. What is the probability that **that** person is female? [Hint: start with a tree diagram.]

- (a)  $\frac{(.45)(.95)}{(.45)(.95) + (.45)(.05)}$       (b)  $\frac{(.45)(.95)}{(.55)(.85)}$       (c) .45
- (d)  $(.45)(.95)$       (e)  $\frac{(.45)(.95)}{(.45)(.95) + (.55)(.85)}$

8. (5 pts.) In randomly chosen group of 15 people, what is the probability that **at least** two have the same birthday?

- (a)  $\frac{C(15, 2)}{365^{15}}$       (b)  $\frac{C(15, 2)(363)(362) \cdots (351)}{365^{15}}$
- (c)  $1 - \frac{(365)(364) \cdots (351)}{365^{15}}$       (d)  $1 - \frac{C(15, 2)}{365^{15}}$
- (e)  $1 - \frac{C(15, 2) \cdot C(363, 2)}{365^{15}}$

9. (5 pts.) I have a deck of 24 cards consisting of the  $A$ , 2, 3, 4, 5 and 6 of each suit. Bill, Claire, Doug and Emily are seated around a table. From this deck the dealer gives 2 cards to Bill, two cards to Claire, two cards to Doug and two cards to Emily (**without replacement**). What is the probability that **none** of them gets an ace? [Hint: of the 24 cards, four are aces and 20 aren't.]

- (a)  $\frac{P(20, 2) \cdot P(18, 2) \cdot P(16, 2) \cdot P(14, 2)}{P(24, 8)}$       (b)  $1 - \frac{P(20, 8)}{P(24, 8)}$
- (c)  $\frac{P(24, 2) \cdot P(22, 2) \cdot P(20, 2) \cdot P(18, 2)}{P(24, 8)}$       (d)  $1 - \frac{4 \cdot P(20, 2)}{P(24, 8)}$
- (e)  $\frac{4 \cdot P(20, 2)}{P(24, 8)}$

10. (5 pts.) You are dealt 13 cards from a standard deck of 52 cards (without replacement). What is the probability that you get both the Jack of diamonds and the Queen of spades?

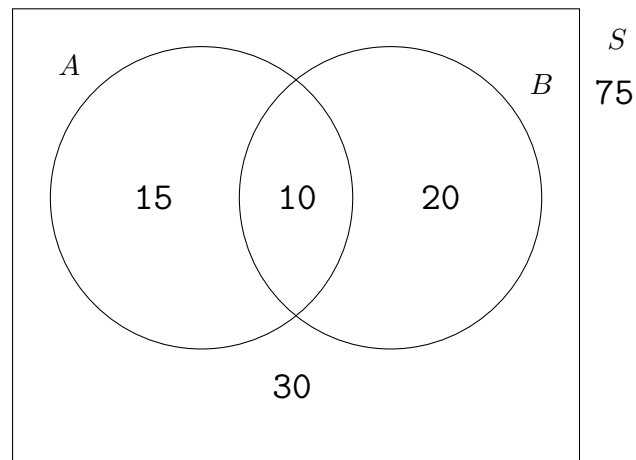
- (a)  $\frac{C(52, 11)}{C(52, 13)}$       (b)  $\frac{P(50, 11)}{C(52, 13)}$       (c)  $\frac{C(13, 2) \cdot C(39, 11)}{C(52, 13)}$
- (d)  $\frac{C(50, 11)}{C(52, 13)}$       (e)  $\frac{2 \cdot C(50, 11)}{C(52, 13)}$

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.)

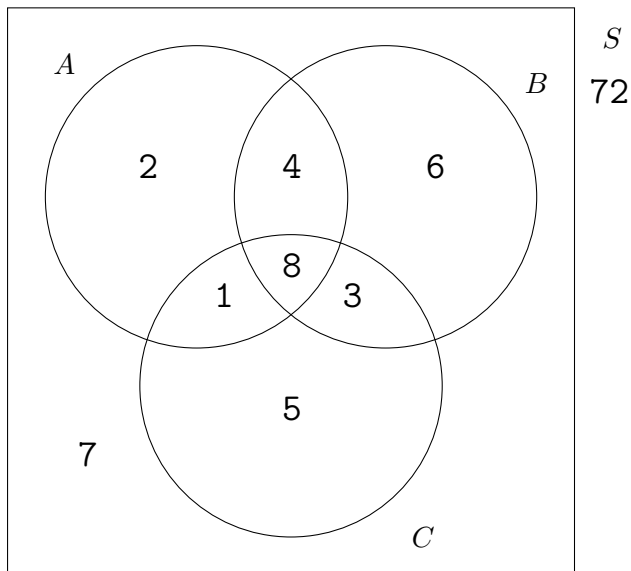
The following Venn diagram describes the relative sizes of events  $A$ ,  $B$  and  $C$  in a sample space  $S$ .



(a) Show that events  $A$  and  $B$  are independent. You have to fully explain your answer to get credit.

(b) Are events  $A$  and  $B$  mutually exclusive? Give a clear explanation of your answer.

12. (10 pts.) The following Venn diagram describes the relative sizes of events  $A, B$  and  $C$  in a sample space  $S$ .



Find each of the probabilities using the numbers in the diagram. For example, if I asked for  $P(A)$ , I'd like you to write

$$\frac{2 + 4 + 8 + 1}{72}. \quad \text{You would not have to write } = \frac{15}{72} = \frac{5}{24} \text{ unless you want to.}$$

(Any order of the numbers in the numerator would be fine.)

(a)  $P(A \cap B) =$

(b)  $P(C \mid A) =$

(c)  $P(A \cap B \cap C') =$

(d)  $P(A \cap B \mid C) =$

(e)  $P(C \mid A \cap B) =$



**13.** (10 pts.) (These numbers are totally fictitious.) In the city of Philadelphia, 99% of the people are Eagles fans and 1% are not.

- Of the Eagles fans, 98% own an Eagles sweatshirt and 2% do not.
- Of the non-Eagles fans, 5% own an Eagles sweatshirt and 95% do not.

(a) Draw a tree diagram representing this situation. Explain your notation and be sure to include the probabilities.

(b) If a person in Philadelphia is chosen at random, find the probability that she owns an Eagles sweatshirt. Show your work!!!

(c) A person in Philadelphia is chosen at random, and it is determined that he does not own an Eagles sweatshirt. Find the probability that he is an Eagles fan. Show your work!!!

**14.** (10 pts.) A bag contains 9 colored marbles, of which 5 are red and 4 are blue. I plan to pick 3 marbles from the bag **without replacement**.

**Note:** In the following two parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ). Be sure to mark your answer.

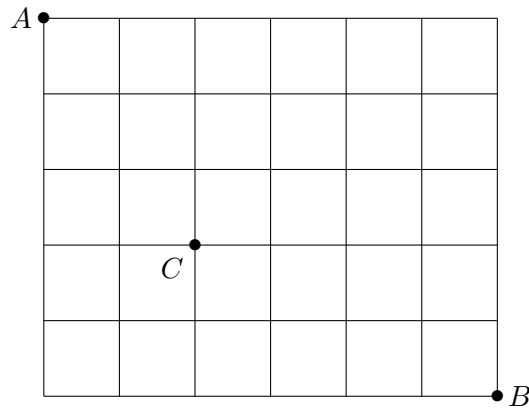
(a) Assuming that order is not important, what is the probability that all three are the same color?

(b) Assuming that order **is** important, what is the probability that the first is red, then the second is blue, then the third is red. [Don't forget that this is done **without** replacement.]

**15.** (10 pts.) In this problem, be sure to show all your work and be sure to plainly mark your answer. The following is a street map of part of a city. Emily lives at the northwest corner (marked  $A$ ) and wants to get to the library at the southeast corner (marked  $B$ ). She decides to take an Uber. The driver travels only east and south (i.e. to the right or down), following the roads and randomly choosing between east and south at each intersection he comes to. What is the probability that the Uber will pass by Claire's house, marked  $C$ ?

[Hint: how many routes are there to get to the library, and of those how many pass by  $C$ ?]

**Note:** In this problem, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ( $P(n, k)$ ), combinations ( $C(n, k)$ ), factorials ( $n!$ ) and powers ( $a^k$ ).



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MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

Tot. \_\_\_\_\_