Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math

Spring, 2020

Name: Solutions

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Practice Exam 3b

This exam is in two parts on 12 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
12.	________
13.	________
14.	________
15.	________
Tot.	________

Multiple Choice

1. (5 pts.) When Father Jenkins and Provost Burish play tennis, they play three sets. Experience suggests that Fr. Jenkins will win all three sets with probability 1/8, he will win exactly one set with probability 1/4, and he will win none of the sets with probability 1/4. What is the mean (expected) number of sets that Fr. Jenkins will win?

(a) $\frac{11}{8}$ (b) $\frac{3}{2}$ (c) $\frac{13}{8}$

(d) 1 (e) 2

X = # of games won

k	P(X=k)
0	1/4
1	1/4
2	1−1/4−1/4−1/8 = 3/8
3	1/8

$E[X] = 0\times\frac{1}{4} + 1\times\frac{1}{4} + 2\times\frac{3}{8} + 3\times\frac{1}{8}$

$= \frac{11}{8}$

2. (5 pts.) We play this game: you roll an ordinary dice. If the dice shows "1", I give you \$12. If the dice shows "2" or "3", I give you \$9. If the dice shows "4", "5" or "6", I give you \$6. Let X be the amount you win when playing this game. What is the *standard deviation* of X?

(a) $\sqrt{61}$ ~~(b)~~ $\sqrt{5}$

(c) 17 (d) $\sqrt{17}$

(e) 5

K	P(X=k)	k·P(X=k)	(k−E[X])²	(k−E[X])²·P(X=k)
12	1/6	12·1/6 = 2	(12−8)² = 16	16·1/6 = 8/3
9	1/3	9·1/3 = 3	(9−8)² = 1	1·1/3 = 1/3
6	1/2	6·1/2 = 3	(6−8)² = 4	4·1/2 = 2
		E[X] = sum = 8		Var(X) = 5 = σ²(X)

σ(X) = $\sqrt{5}$

3. (5 pts.) The probability that the basketball player Kevin Durant makes a free throw is 85%. If he shoots 15 free throws in a game, what is the probability that he makes 13 or more free throws?

(a) $(0.85)^{13}+(0.85)^{14}+(0.85)^{15}$

(b) $C(15,13)(0.85)^{13}+15(0.85)^{14}+(0.85)^{15}$

(c) $C(15,13)(0.85)^{13}(0.15)^{2}$

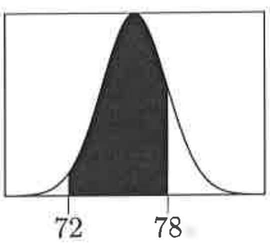
(d) $C(15,13)(0.85)^{13}$

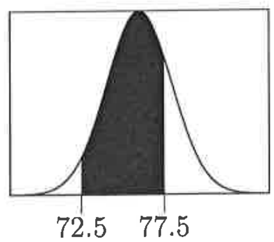
(e) $C(15,13)(0.85)^{13}(0.15)^{2}+15(0.85)^{14}(0.15)^{1}+(0.85)^{15}$

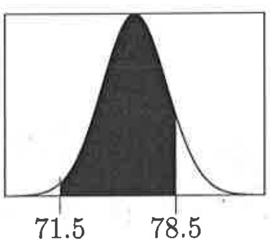
Binomial with $n=15$, $p=0.85$

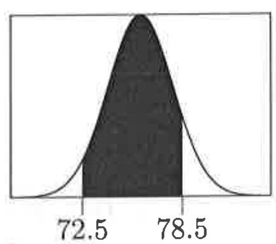
$$\binom{15}{13}(0.85)^{13}(0.15)^{2}+\binom{15}{14}(0.85)^{14}(0.15)^{1}+\binom{15}{15}(0.85)^{15}(0.15)^{0}$$

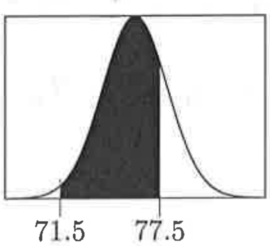
4. (5 pts.) A certain vaccine is administered to 80 patients. The probability that the vaccine will prevent a certain disease is 95%. The area of which of the following shaded regions gives a better estimate for the probability that between 72 and 78 (inclusive) of them will not get infected by the disease?

(a)


(b)


(c)


(d)


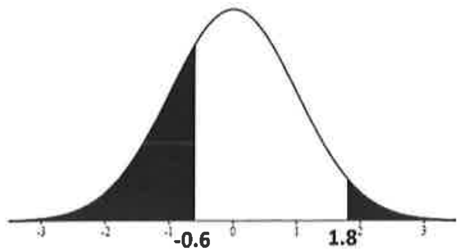
(e)


X is Binomial with $n=80$, $p=0.95$

N is Normal with $\mu=76$, $\sigma\approx 1.95$

$P(72\le X\le 78)\approx P(71.5\le N\le 78.5)$

5. (5 pts.) The picture below shows the standard normal probability curve ($\mu = 0$, $\sigma = 1$). What is the area of the black region? [Answers are all rounded to nearest percent.]



(a) 69% (b) 27%

(c) 38% (d) 4%

(e) 31%

$1 - P(-0.6 \le N \le 1.8)$

$= 1 - (A(0.6) + A(1.8))$

$= 1 - (0.2258 + 0.4641)$

$= 0.3101 \approx 31\%$

6. (5 pts.) John plans to spend time in Ireland after final exam week, and he plans to visit Galway and Kerry. He has at most 7 days available and at most 500 euros to spend. Each day spent in Galway will cost 50 euros, and each day spent in Kerry will cost 60 euros. Let x be the number of days that John will spend in Galway and let y be the number of days that he will spend in Kerry. Which of the following inequalities properly describe all constraints on John's time and money?

(a) $x + y \le 7$, $50x + 60y \le 1000$

(b) $x + y \le 7$, $50x + 60y \le 500$, $x \ge 0$, $y \ge 0$

(c) $x + y \ge 7$, $60x + 50y \ge 500$, $x \ge 0$, $y \ge 0$

(d) $x + y \ge 7$, $50x + 60y \ge 500$

(e) $x + y \le 7$, $60x + 50y \le 500$, $x \ge 0$, $y \ge 0$

7. (5 pts.) Which of the following points is in the feasible region for the following system of inequalities (note that the first constraint has a $<$ rather than a $\leq$)

$$3x - 2y < 4$$
$$8x + 4y \leq 24$$
$$x \geq 0, y \geq 0$$

(a) $(1, -2)$ (b) $(2, 1)$ (c) $(-1, -2)$

(d) $(2, 2)$ (e) $(1, 5)$

8. (5 pts.) Let Z be a random variable corresponding to the standard normal curve. Find the value of b so that $Pr(0.5 \leq Z \leq b) = 0.2590$. Use the table at the end of this exam.

(a) $b = 1.65$ (b) $b = 0.65$ (c) $b = 0.5325$

(d) $b = 0.75$ (e) $b = 1.55$

$Pr(0.5 \leq Z \leq b) = \cancel{A(0.5)}\ A(b) - A(0.5)$

$A(b) - A(0.5) = 0.2590$

$\cancel{A(b) - 0.6915 = 0.2590}$

$\cancel{A(b) = 0.9505}$

$\cancel{b = 1.65}$

$A(b) - 0.1915 = 0.2590$

$A(b) = 0.4505$

$b = 1.65$

9. (5 pts.) Let X be a random variable associated to a normal curve with mean $\mu = 25$ and standard deviation σ. If $Pr(X \leq 30) = 97.72\%$, find the value of σ.

(a) $\sigma = 3$ (b) $\sigma = 1$ (c) $\sigma = 1.5$ (d) $\sigma = 2.5$ (e) $\sigma = 2$

$P(X \leq 30) = 0.9772$

$Pr\left(Z \leq \frac{30-25}{\sigma}\right) = 0.9772$
$= 0.5 + 0.4772$
$= 0.5 + A(2.0)$

so $\frac{30-25}{\sigma} = 2.0$

$2\sigma = 5$
$\sigma = 2.5$

10. (5 pts.) Assume that the heights of NBA players are normally distributed, with a mean of 80 inches, and a standard deviation of 4 inches. If an NBA player is chosen at random, what is the probability that he is at least 85 inches tall? Use the table at the end of this exam.

(a) 9.28% (b) 8.72% (c) 0% (d) 89.44% (e) 10.56%

$P(X \geq 85) = 1 - P(x \leq 85)$

$= 1 - P\left(Z \leq \frac{85-80}{4}\right) = 1 - P(z \leq 1.25)$

$= 0.5 + A(1.25) = 0.5 + 0.3944$
$= 0.8944$

Partial Credit

You must show **all of your work** on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The following are golf scores from 10 rounds of golf for two players, Player A and Player B:

Player A:	91, 95, 96, 94, 95, 92, 93, 99, 97, 98
Player B:	96, 95, 97, 94, 95, 94, 93, 95, 96, 95

For both players, the average (mean) score is 95.

(a) For A, these are the only ten rounds of golf he will ever play. Calculate the (population) variance σ^2 of A's scores.

$$\sigma^2 = \frac{(91-95)^2 + (95-95)^2 + \dots + (98-95)^2}{10}$$

$$= 6$$

*<u>Only</u> whole numbers

(b) What is the z-score of A's round of 92?

$$z\text{-score is } \frac{92-95}{\sqrt{6}} \approx -1.22$$

(c) For B, these scores are just a sample from the large number of rounds that she has played/will ever play. Calculate the (sample) variance s^2 of B's scores.

$$s^2 = \frac{(96-95)^2 + (95-95)^2 + \dots + (95-95)^2}{9}$$

$$\approx 1.33$$

(d) What are the possible round scores for B that fall within two-and-a-half (sample) standard deviations of her (sample) mean?

$s = 1.1547$, so $2\frac{1}{2}$ sample standard deviations is ≈ 2.88

The possible whole numbers in range 95 ± 2.88 are

93, 94, 95, 96, 97 *

12. (10 pts.) An experiment randomly selects two people from a group of 8 men and 5 women. Let X be the number of women selected.
(a) List the possible values of X.

0, 1, 2

(b) Compute the probability distribution of the random variable X. You can either draw a table with the required values or draw a histogram.

X	$P(X)$
0	$C(8,2)/C(13,2) = .36$
1	$\dfrac{C(8,1)C(5,1)}{C(13,2)} = .51$
2	$\dfrac{C(5,2)}{C(13,2)} = .13$

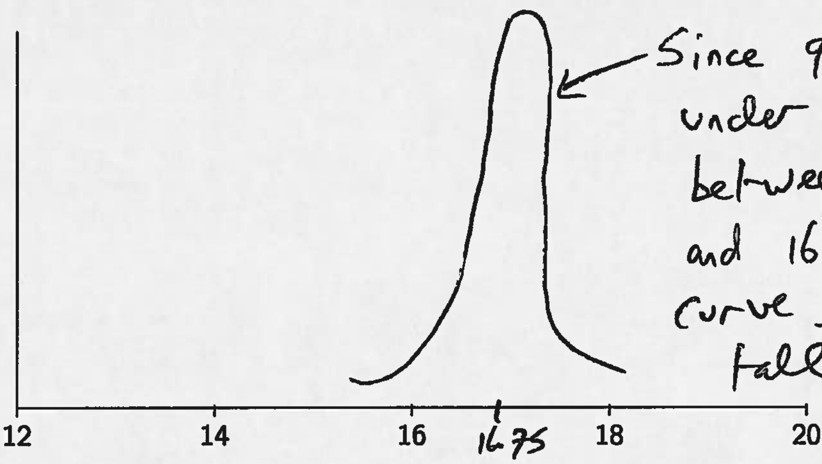
[NOT $\left(\frac{8}{13}\right)^2$; once first person has been selected, pool for second selection has changed]

(c) Compute the expected value of X, that is, compute $E(X)$.

$$E(X) = 0(.36) + 1(.51) + 2(.13)$$
$$= .77$$

13. (10 pts.) At a soft drink bottling plant, the amount of cola put into bottles is normally distributed with $\mu = 16.75\ oz$ and $\sigma = 0.5\ oz$. Let X be the amount of cola put into a bottle.

(a) On the axes below, sketch the normal curve of the random variable X.

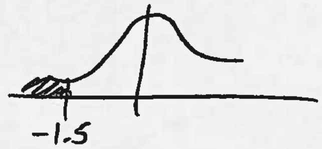


Since 95% of area under curve should be between 16.75 − 2(.5) and 16.75 + 2(.5), curve should be quite tall and thin

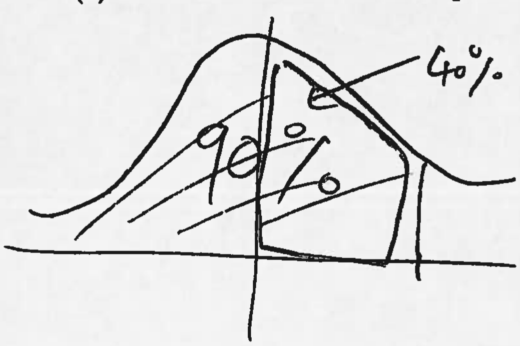
(b) What is the probability that a bottle will contain less than 16 oz of cola?

16 oz has z-score $\dfrac{16-16.75}{.5} = -1.5$

From table, $P(z\text{-score} \le -1.5) = .0668$



(c) For what value x will the probability that a bottle will contain less than x oz of cola be 90%?



40% in here

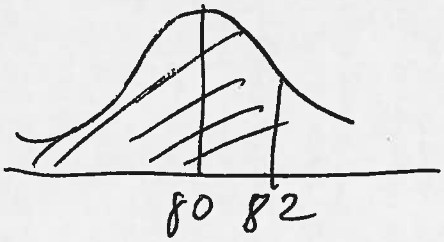
From table,

$P(z\text{-score} \le 1.28) = .9$

So x should have z-score of 1.28

$1.28 = \dfrac{x - 16.75}{.5}$, $x = 17.39$

14. (10 pts.) One hour after iron rods come out of a tempering chamber, their temperatures are normally distributed with mean 80 degrees, and variance 25.
(a) What is the probability that a rod is cooler than 82 degrees one hour after it comes out of a tempering chamber?



z-score of 82 is $\frac{82-80}{5} = .4$

From table,

$P(\text{z-score} \leq .4) = .6554$

Consider the following Bernoulli experiment:
An engineer took 30 rods out of a tempering chamber one hour ago. He can only use a rod if it is cooler than 82 degrees (notice that in part (a), you computed the probability that a rod is cooler than 82 degrees one hour after it comes out of the chamber).

(b) Use the normal approximation to the binomial to estimate the probability that the engineer will be able to use at least 20 of the rods.

X = # good rods = Binomial with $n = 30$, $p = .6554$

mean = np = 19.662

std dev = $\sqrt{npq}$ = 2.603

Want: $P(X \geq 20)$

Continuity correction

Approximate with $P\left(\begin{matrix}\text{Normal, mean } 19.662 \\ \text{std dev } 2.603\end{matrix} \geq 19.5\right)$

Using normal calculator, get probability

$\approx .5248$

15. (10 pts.) A farmer has 100 acres on which to plant oats or corn. Each acre of oats requires 2 hours of labor and each acre of corn requires 6 hours of labor. He has 300 hours of labor available. The revenue he obtains is \$55 for each acre of oats and \$125 for each acre of corn. The farmer wants to maximize his revenue.

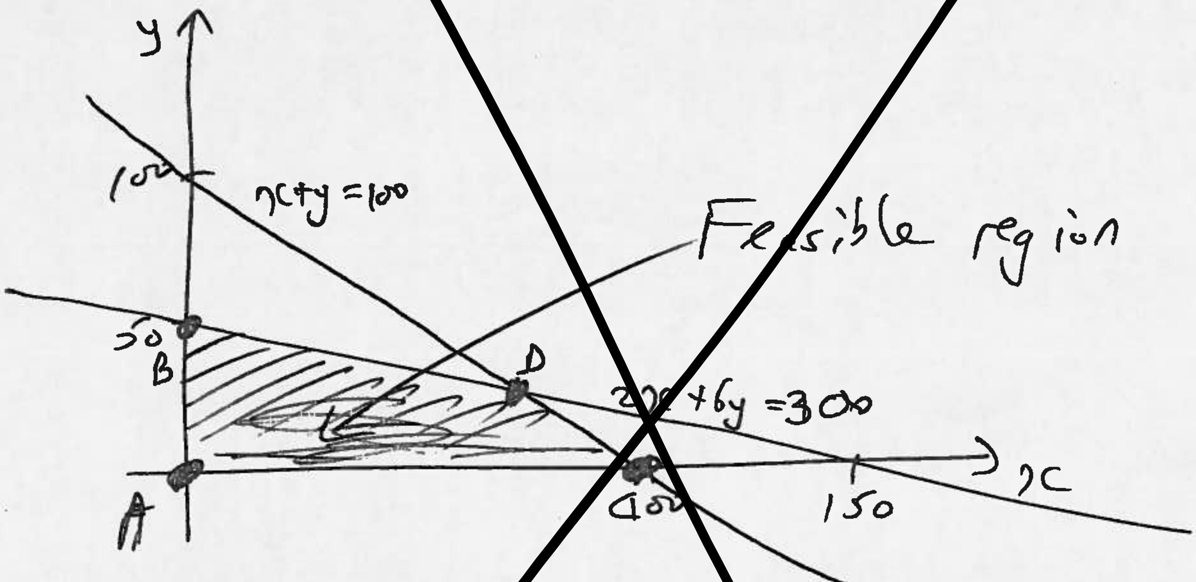
(a) Write the constraints and objective function.

x = # acres oats
y = # acres corn

Objective: Maximize $55x + 125y$

Constraints: $x \geq 0$, $y \geq 0$, $x + y \leq 100$, $2x + 6y \leq 300$

(b) Draw the feasible region of the system of constraints. Mark the feasible region clearly.



(c) Make a table with two columns, one with the corner points and the other with value of the objective function at the given corner point.

Corner point	objective
A ($x=0$, $y=0$)	0
B ($x=0$, $y=50$)	6250
C ($x=100$, $y=0$)	5500
D ($x=75$, $y=25$)	7250 ← largest

(d) What planting combination will produce the greatest total revenue?

75 acres of oats,
25 acres of corn