| Name: | | | |
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Instructor:_____

Section number:_____

Department of Mathematics University of Notre Dame Math 10250 – Elem. of Calc. I Fall 2022

Exam 1

September 15, 2022

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. No books, notes, phones or other aids are permitted. Be sure to write your name on this title page, and in case pages become detached put your initials at the top of each.

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You must record here your answers to the multiple choice problems by placing an \times through your answer to each problem.

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Initials:_____
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Multiple Choice

1. (5 pts.) Suppose that f and g are functions such that f(2) = 3, f(-3) = -2, g(3) = 4 and g(2) = -3. What is the value of $(f \circ g)(2)$?

- (a) -2
- (b) -3
- (c) -9
- (d) 4
- (e) 3

2. (5 pts.) Consider the graph of a function f(x) given below:



Which of the following is true?

- (a) At x = 2, f(x) is both continuous and differentiable.
- (b) At x = 2, f(x) is differentiable but not continuous.
- (c) At x = 2, f(x) is neither differentiable nor continuous.
- (d) At x = 2, f(x) is continuous but not differentiable.
- (e) $\lim_{x \to 2} f(x)$ is undefined.

3. (5 pts.) Which of the following expressions is equal to

 $\frac{x-1}{x+2} - \frac{x+2}{x-1}?$

- (a) $-\frac{6x+3}{(x-1)(x+2)}$ (b) $\frac{2x+1}{3}$
- (c) $-\frac{2x^2+2x+5}{(x-1)(x+2)}$
- (d) $\frac{6x-3}{(x-1)(x+2)}$

(e)
$$-\frac{2x+1}{3}$$

4. (5 pts.) Mary has 100 feet of fencing and wants to make a rectangular enclosure. If the width of the rectangle is x, what function measures the area of the rectangle? (*Hint: as a first step you'll have to find the length as a function of x*.)

- (a) $100x x^2$
- (b) $50x 2x^2$
- (c) $50x x^2$
- (d) $x^2 50x$
- (e) There is not enough information to answer the question.

- **5.** (5 pts.) Suppose $f(x) = x^2 + 1$ and $g(x) = x^2 1$. Find $(f \circ g)(x)$
- (a) $(f \circ g)(x) = x^4 2x^2 + 3$
- (b) $(f \circ g)(x) = x^4 2x^2 + 2$
- (c) $(f \circ g)(x) = x^4 1$
- (d) $(f \circ g)(x) = x^4 + 2x^2 + 1$
- (e) $(f \circ g)(x) = x^4 + 2x^2$

6. (5 pts.) Suppose that the function

$$h(t) = \frac{125t^2}{5t^2 + 25t + 100}$$

describes the height (in meters) of a certain species of tree after t years. The tree can live for centuries. What is the approximate height of a very old member of this species? (*Hint: think about the limit as t approaches infinity.*)

- (a) 5 meters
- (b) 0 meters
- (c) 125 meters
- (d) 25 meters
- (e) The tree grows arbitrarily high if you wait long enough.

7. (5 pts.) Let f(x) be the function

$$f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x - 1} & \text{if } x \neq 1 \\ c & \text{if } x = 1. \end{cases}$$

Which value of c will make f(x) continuous on all real numbers?

- (a) -2
- (b) 3
- (c) = 0
- (d) There is no such c.
- (e) -1

8. (5 pts.) Let f(x) be a function such that f(0) = 0, f(2) = 10 and f'(2) = 3. Which of the following is **FALSE**?

- (a) The average rate of change of f(x) on [0, 2] is 3.
- (b) The slope of the tangent line to f(x) at x = 2 is 3.
- (c) f(x) is continuous at x = 2.
- (d) The instantaneous rate of change of f(x) at x = 2 is 3.

(e)
$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 3$$

Initials:_____

Initials:_____

9. (5 pts.)

The slope of the tangent line to $f(x) = \frac{x^3}{16} + 2\sqrt{x} + 27$ at x = 4 is

- (a) 3
- (b) 4
- (c) 3.5
- (d) = 0
- (e) -3

10. (5 pts.) Find h'(2) when h(x) = f(x)g(x), f(2) = -1, $f'(2) = \frac{1}{3}$, g(2) = 3, and g'(2) = 5.

- (a) 0
- (b) $-\frac{5}{3}$
- (c) -4
- (d) 2
- (e) There is not enough information given

Initials:_____

11. (5 pts.)

Evaluate the following limit:

$$\lim_{h \to 0} \frac{2(1+h)^3 - 2}{h}$$

(a) 6

- (b) -2
- (c) 2
- (d) 7
- (e) The limit does not exist.

Initials:_____

Partial Credit

You must show your work on the partial credit problems to receive credit!

12. (15 pts.) (Again, be sure to show all work.)

(a) What is the slope of the line passing through the points (1, 2) and (3, -1)?

(b) What is the slope of a line which is perpendicular to the line in (a)?

(c) Find the equation of the line which passes through the point (1, 1) and is perpendicular to the line in (a). Put your final answer for the equation of the line in slope-intercept form (y = mx + b).

13. (15 pts.)

Consider the function $f(x) = 5x^2$.

(a) Use the limit definition to compute the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

by completing the following steps:

(i) Compute f(x+h).

(ii) Compute the difference quotient $\frac{f(x+h) - f(x)}{h}$.

(iii) Compute the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \; .$$

(b) What is the derivative of $5x^2 + 120$? (*Hint*: Use derivative rules. No need to compute a limit.) Initials:_____

Initials:_____

14. (15 pts.) Let

$$f(x) = \frac{x^2 - 4x + 7}{2x - 1}.$$

(a) Find the derivative of f(x). There is no need to simplify.

(b) Find the equation of the tangent line to the graph of y = f(x) at x = 1.

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