

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I
Fall 2022

Name:_____

Instructor:_____

Time MWF class meets:_____

Exam 2

October 11, 2022

This exam is in 2 parts on 10 pages and contains 12 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. **No books, notes, phones or other aids other than calculators are permitted.** Be sure to write your name on this title page, and in case pages become detached put your initials at the top of each.

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Signature:_____

You must record here your answers to the multiple choice problems by placing an $\times$ through your answer to each problem.

- | | | | | | |
|----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
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MC. _____
9. _____
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Tot. _____

Multiple Choice

1. (5 pts.) The cost of making x supercars is $C(x) = 2\sqrt{x^2 - 16}$ million dollars. What is the marginal cost (in millions of dollars) when 5 cars are produced?

- (a) 5
- (b) 6
- (c) $\frac{5}{3}$
- (d) 3
- (e) $\frac{10}{3}$

2. (5 pts.) $y = f(u)$ and $u = h(x)$. What is $\frac{dy}{dx}$ when $x = 2$, given that

$$h(2) = 3, \quad f(2) = 4, \quad h'(2) = 5, \quad h'(3) = -5, \quad f'(2) = 6, \quad f'(3) = 7, \quad \text{and} \quad f'(4) = 8?$$

- (a) 30
- (b) 35
- (c) 40
- (d) 6
- (e) -30

3. (5 pts.) The volume V in cubic centimeters of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere in centimeters.

A child who is blowing up a spherical balloon accidentally lets it go. A few moments after it is released, the balloon has a radius of 10 cm and the air pressure inside the balloon is forcing the air out through the balloon's opening at a rate of 4,000 cubic centimeters per second. At what rate is the radius of the balloon decreasing at that time? (Assume the balloon remains spherical while deflating.)

- (a) $10/\pi$ centimeters per second
- (b) $1/\pi$ centimeters per second
- (c) $5/\pi$ centimeters per second
- (d) $30/\pi$ centimeters per second
- (e) $20/3\pi$ centimeters per second

4. (5 pts.) Let $f(x) = \sqrt{x}$. Which of the following is the second derivative $f''(x)$ of $f(x)$?

- | | |
|------------------------------------|------------------------------------|
| (a) $\frac{1}{4}x^{-\frac{3}{2}}$ | (b) $-\frac{1}{2}x^{-\frac{1}{2}}$ |
| (c) $\frac{1}{2}x^{-\frac{1}{2}}$ | (d) $\frac{3}{8}x^{-\frac{5}{2}}$ |
| (e) $-\frac{1}{4}x^{-\frac{3}{2}}$ | |

5. (5 pts.) Suppose $f(x)$ is a function and you know the following information:

- $f(x)$ is defined for all x .
- $f'(x) = 0$ for $x = 1$ and $x = 5$ but no other x .
- $f'(0) = -4$, $f'(3) = 6$ and $f'(7) = -1$.

Which of the following is **TRUE**? (Hint: draw a number line for the sign of $f'(x)$.)

- (a) $f(x)$ has a relative max at $x = 1$, and a relative min at $x = 5$.
- (b) $f(x)$ has a relative min at both $x = 1$ and $x = 5$.
- (c) $f(x)$ has a relative min at $x = 1$, and a relative max at $x = 5$.
- (d) $f(x)$ has a relative max at both $x = 1$ and $x = 5$.
- (e) $f(x)$ is a constant function.

6. (5 pts.) Let $f(x) = 3x^2 - 6x + 4$. Which of the following is **TRUE** about $f(x)$?

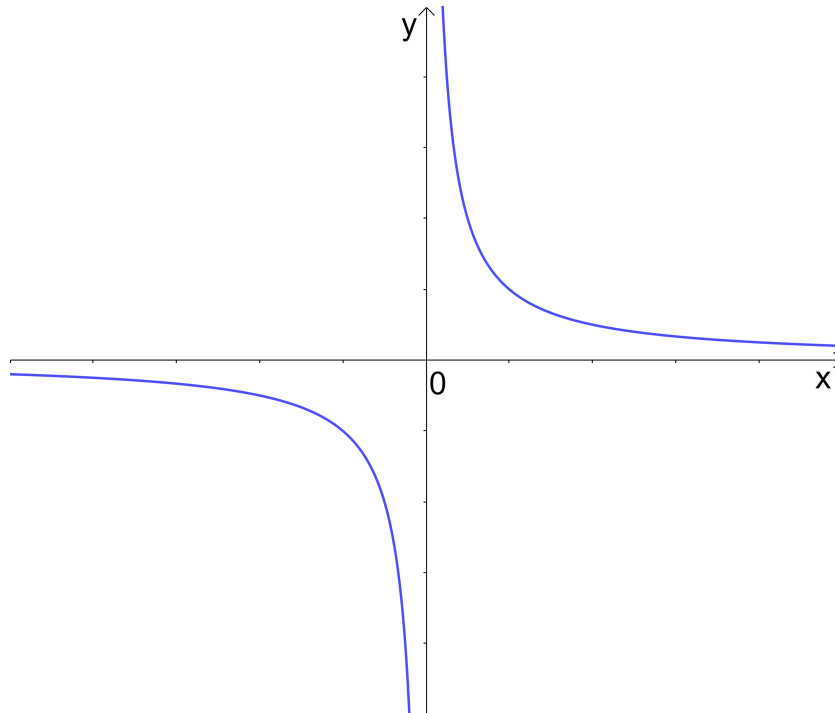
- (a) $f(x)$ has a relative min at $(1, 1)$ and is concave down at that point.
- (b) $f(x)$ has a relative min at $(1, 0)$ and is concave up at that point.
- (c) $f(x)$ has a relative min at $(1, 1)$ and is concave up at that point.
- (d) $f(x)$ has a relative min at $(1, 0)$ and is concave down at that point.
- (e) $f(x)$ has a relative max at $(1, 1)$ and is concave down at that point.

7. (5 pts.) Find the horizontal and vertical asymptotes of the function

$$f(x) = \frac{4x}{x^2 - x - 6}$$

- (a) Horizontal: $y = 4$; Vertical: $x = 2$ and $x = -3$.
- (b) Horizontal: $y = 4$; Vertical: None
- (c) Horizontal: $y = 2$ and $y = -3$; Vertical: $x = 0$.
- (d) Horizontal: $y = 0$; Vertical: $x = -2$ and $x = 3$.
- (e) Horizontal: $y = 0$; Vertical: $x = 0$.

8. (5 pts.) The graph of a function $f(x)$ is given below.



Which of the following statements is **FALSE**?

- (a) $f'(x) > 0$ for $x > 0$.
- (b) $f(x)$ has a vertical asymptote at $x = 0$.
- (c) $f(x)$ is concave down for $x < 0$.
- (d) $f''(x) > 0$ for $x > 0$.
- (e) $f'(x) < 0$ for $x < 0$.

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

9. (15 pts.)

The cost of making bikes is $C(x) = 6000 + 200x - x^2$. The price p depends on x , the number of units produced, by the formula

$$p(x) = 1000 \frac{x+5}{x-5}.$$

- What is the marginal cost when 100 bikes are produced?

- What is the average cost function, $\overline{C}(x)$?

- What is the marginal average cost when 100 bikes are produced?

- What is the revenue function?

- What is the profit function?

10. (15 pts.) Consider the curve defined implicitly by the equation $x^3 + 3x + y^3 + 3y = 8$.

(a) By differentiating the equation implicitly, find an expression for $\frac{dy}{dx}$ in terms of x and y .

(b) Explain clearly why the point $(1, 1)$ is on the curve.

(c) Determine the equation of the tangent line to the curve at the point $(1, 1)$.

11. (15 pts.) Let $f(x)$ be some function whose **derivative** is

$$f'(x) = \frac{x+1}{(x-1)(x-3)}.$$

(Note this is already f' – you don't have to differentiate it.) Assume also that the original function $f(x)$ has a vertical asymptote (in particular is undefined) at $x = 1$ and at $x = 3$ but is defined everywhere else.

- (a) Tell us on what intervals $f(x)$ is increasing and on what intervals it is decreasing. Be sure to show all necessary work – you must give some justification to get full credit.

- (b) Using your results from (a) and taking into account the asymptotes, identify the values of x , if any, where $f(x)$ has relative maxima and relative minima.

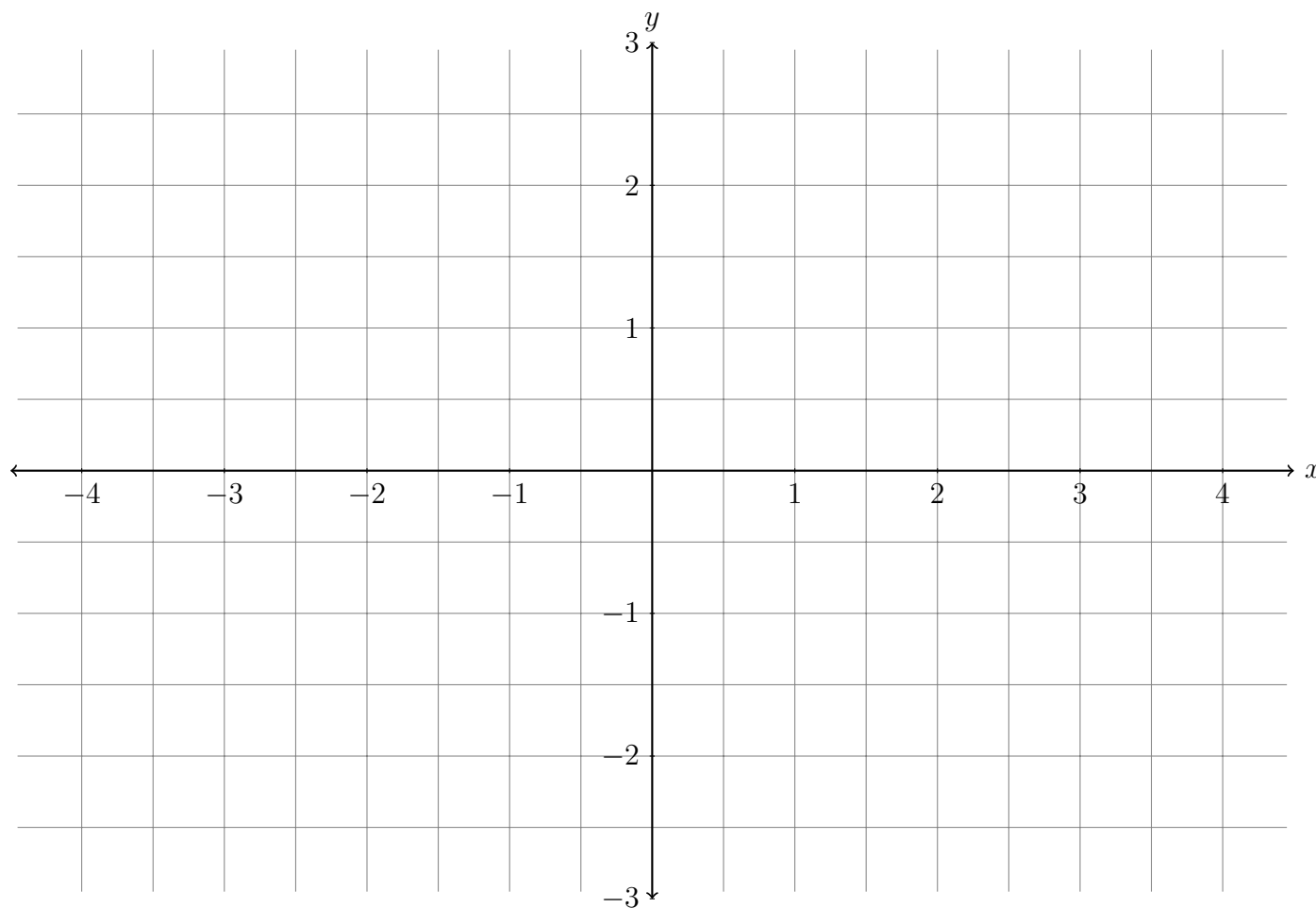
12. (15 pts.) Draw the graph of a function $y = f(x)$ satisfying the following list of properties. Specify the coordinates (x, y) of any relative minima, relative maxima, and inflection points in the blanks below (write NONE if there are none). Draw any asymptotes with a dotted line.

- The domain is all real numbers.
- $f(-2) = 0$, $f(0) = 2$, and $f(2) = 0$.
- $\lim_{x \rightarrow -\infty} f(x) = -1$
- $\lim_{x \rightarrow \infty} f(x) = -1$
- $f'(x) < 0$ on $(0, \infty)$.
- $f'(x) > 0$ on $(-\infty, 0)$.
- $f'(0) = 0$.
- $f''(x) < 0$ on $(-2, 2)$.
- $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$.

Inflection point(s) at _____

Relative maxima at _____

Relative minima at _____



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