

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I
Fall 2022

Name:_____

Instructor:_____

Time MWF class meets:_____

Exam 3

November 17, 2022

This exam is in 2 parts on 9 pages and contains 12 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. **No books, notes, phones or other aids other than calculators are permitted.** Be sure to write your name on this title page, and in case pages become detached put your initials at the top of each.

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:_____

You must record here your answers to the multiple choice problems by placing an \times through your answer to each problem.

- | | | | | | |
|----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
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Multiple Choice

1. (5 pts.) What is the maximum value for $f(x) = x^3 - 3x^2$ in the interval $[0, 4]$. (Notice this same function and interval appears in problem 2 as well. The other time it asks for the minimum value.)

- (a) 5
- (b) 0
- (c) -4
- (d) 16
- (e) 1

2. (5 pts.) What is the minimum value for $f(x) = x^3 - 3x^2$ in the interval $[0, 4]$. (Notice this same function and interval also appears in problem 1. The other time it asks for the maximum value.)

- (a) -4
- (b) 5
- (c) 0
- (d) 16
- (e) 1

3. (5 pts.) Which of the following real numbers x satisfies the equation

$$\frac{e^x}{1 - e^x} = 2?$$

- (a) $x = \ln 3 - \ln 4$
- (b) $x = -1$
- (c) $x = \ln 2$
- (d) $x = \ln 2 - \ln 3$
- (e) $x = \ln 3$

4. (5 pts.) Use logarithmic differentiation to find the derivative of the function $f(x) = x^{2x}$.

- (a) $2x \cdot x^{2x-1}$
- (b) $x^{2x} \ln(2x)$
- (c) $(2 \ln x + 2)x^{2x}$
- (d) $(\ln x + 1)x^{2x}$
- (e) $2 \ln x \cdot x^{2x}$

5. (5 pts.) Bob's family has a tradition that upon finishing college, the graduate gives his or her parents a check for \$10,000. Bob plans to graduate from Notre Dame in May of 2026, so **exactly four years** in advance (May, 2022) he decided to invest some sum of money so that in four years he would have exactly \$10,000 in the account. He found an account bearing 5% interest, compounded continuously. How much should he have invested in May, 2022, in order to fulfill his plans of reaching \$10,000 in four years?

- (a) \$12,214.03
- (b) \$9,512.29
- (c) \$2,500.00
- (d) \$8,227.02
- (e) \$8,187.31

6. (5 pts.) Find the critical numbers for the function $f(x) = x^3e^{-x}$ (i.e. find the values of x where $f'(x)$ is either undefined or equal to 0).

- (a) $x = 0$ only
- (b) $x = 0, 3$
- (c) $x = 3$ only
- (d) $x = 0, 1, 3$
- (e) $x = 1$ only

7. (5 pts.) Find the equation of the tangent line to the function $f(x) = x^2 \ln(x)$ at $x = 1$.

- (a) $y = x - 1$
- (b) $y = 3x - 3$
- (c) $y = 3x - 2$
- (d) $y = (2e + 1)x - e - 1$
- (e) $y = 2x - 2$

8. (5 pts.) The number of students who contracted the “freshman flu” this year is modeled by

$$Q(t) = \frac{500}{1 + 99e^{-0.8t}}$$

where t is the number of days since the start of the semester. How many students eventually contracted the disease? (That is, what is the limit when t approaches infinity?)

- (a) 50
- (b) 5
- (c) 500
- (d) 100
- (e) 1000

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

- 9.** (15 pts.) We are asked to design a Petri dish with volume $27\pi \text{ cm}^3$ with **minimal** external surface area. Note that a Petri dish has the shape of a cylinder with no top. What are the radius r and height h of the resulting Petri dish? You may use the following formulas:

$$\text{volume} = V = \pi r^2 h, \quad \text{external surface area} = A = \pi r^2 + 2\pi r h.$$

10. (15 pts.)

(a) Evaluate the indefinite integral

$$\int \left(\frac{1}{\sqrt{x}} - \frac{3}{x} + 2x - 2 + e^x \right) dx.$$

(b) Solve the initial value problem

$$f'(x) = \frac{1}{\sqrt{x}} - \frac{3}{x} + 2x - 2 + e^x, \quad f(1) = e.$$

11. (15 pts.) Sally invests \$5,000 in a bank account with an annual interest rate of 3%. **In all of the following questions, we are looking for the formula.** The numerical answer is not important.

(a) (6 points) If the interest is compounded quarterly (meaning 4 times a year), how much will there be after 10 years?

(b) (6 points) If the interest is compounded continuously, how much will there be after 10 years?

(c) (3 points) In the situation of part (b), write a formula for the effective rate (APR).

12. (15 pts.) Americium-241, a radioactive isotope, is used in smoke detectors. The amount of Americium-241 present in a new smoke detector after t years is modeled by the function

$$Q(t) = 0.3 \left(\frac{1}{2} \right)^{t/432}$$

where the quantity $Q(t)$ is measured in micrograms.

(a) What is the half-life of Americium-241? (That is, how long does it take for the amount present to reduce to half of its initial amount?)

(b) How long will it take for there to be 0.27 micrograms of Americium-241 remaining in the smoke detector?

(c) Find the rate of decay $Q'(t)$. (Hint: you can write $\frac{1}{2}$ as $e^{\ln(\frac{1}{2})}$)

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