| Name: | | |
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Instructor:_____

Section number:_____

Department of Mathematics University of Notre Dame Math 10250 – Elem. of Calculus Fall 2022

Practice Exam 1

September 15, 2022

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. No books, notes, phones or other aids are permitted. Be sure to write your name on this title page, and in case pages become detached put your initials at the top of each.

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:

You must record here your answers to the multiple choice problems by placing an \times through your answer to each problem.

| 1. | (a) | (b) | (c) | (d) | (e) |
|-----|-----|-----|-----|-----|-----|
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |
| 11. | (a) | (b) | (c) | (d) | (e) |

| MC. | |
|------|--|
| 12. | |
| 13. | |
| 14. | |
| Tot. | |

Multiple Choice

1. (5 pts.) The expression

$$\frac{(t^2-1)^{-\frac{1}{2}}}{(t^2-3t+2)^{-\frac{3}{2}}}$$

is equal to which of the following expressions?

(a)
$$\frac{(t-1)^{\frac{1}{2}}(t-2)^{\frac{3}{2}}}{(t+1)^{\frac{1}{2}}}$$

(b)
$$\frac{(t-1)(t-2)^2}{(t+1)^{\frac{1}{2}}}$$

(c)
$$\frac{(t-2)^{\frac{3}{2}}}{(t-1)(t+1)^{\frac{1}{2}}}$$

(d)
$$\frac{(t-2)^{\frac{3}{2}}}{(t-1)^{\frac{1}{2}}(t+1)^{\frac{1}{2}}}$$

1.

(e)
$$\frac{(t-2)^{\frac{3}{2}}}{(t+1)^{\frac{1}{2}}}$$

2. (5 pts.) Read-Til-You-Bleed is a book club. It costs \$60 to be a member, and in addition each member has to pay \$28 for each book they buy. Looks-Like-Books is another book club, which costs \$40 to be a member and each book costs \$32. Frodo wants to join only one of these clubs. How many books would he have to buy so that his total expenses would be equal regardless of which of the two clubs he joins?

- (a)2
- (b) 3
- (c)4
- (d) 5
- (e) 6

Initials:_____

3. (5 pts.) Find the values of x that satisfy the inequality

$$\frac{x+2}{2x-1} < 0.$$

- (a) $-2 < x < \frac{1}{2}$
- (b) $x > -\frac{1}{2}$
- (c) x > -2
- (d) $-\frac{1}{2} < x < 2$
- (e) $x < -2 \text{ or } x > \frac{1}{2}$

4. (5 pts.) Find an equation of the line that passes through the point (1, 2) and is parallel to the line whose equation is

$$-6x + 2y + 12 = 0.$$

- (a) y = -3x
- (b) y = 3x 5
- (c) y = 2x + 3
- (d) y = x 6
- (e) y = 3x 1

5. (5 pts.) In the pair of supply and demand equations below, x represents the quantity demanded and p the unit price. Find the equilibrium price. That is, find the value of p where the supply curve and the demand curve intersect.

Demand equation: $p(x) = \frac{24}{x+4}$, Supply equation: p(x) = x + 2.

- (a)p = 3
- (b) p = 6
- (c) p = 4
- (d) p = 12
- (e) $p = 2\sqrt{3}$

6. (5 pts.) Evaluate the following limit:

$$\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$$

- $\frac{1}{6}$ (a)
- 6(b)
- $\frac{1}{3}$ (c)
- 3 (d)
- (e) The limit does not exist.

7. (5 pts.) Evaluate the limit

$$\lim_{x \to \infty} \frac{3x^2}{2x^2 + 5x - 7}$$

- (a) 0
- (b) $\frac{2}{3}$
- (c) -7
- (d) $\frac{3}{2}$
- (e) The limit does not exist

8. (5 pts.)

Let f(x) be a continuous function and assume that we have the following values for f(x):

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------|---|---|----|----|---|----|---|---|---|
| f(x) | 3 | 5 | -4 | -2 | 1 | -2 | 1 | 5 | 3 |

(For example, this gives f(2) = -4.) On which of the following interval(s) is f(x) guaranteed to take the value 2 at some point? (Warning: we asked for the value 2, not 0.)

- (a) [1, 2], [6, 7]
- (b) [1, 2] only
- (c) [1, 2], [3, 4], [4, 5], [5, 6]
- $(d) \quad [0,1], [1,2], [2,3], [3,4], [4,5], [5,6], [6,7], [7,8]$
- (e) There is no interval where f(x) is guaranteed to take the value 2.

Initials:_____

9. (5 pts.) Let

$$f(x) = x^{\frac{5}{2}}.$$

Find the equation of the tangent line when x = 4.

(a) y - 32 = 4(x - 20)

(b)
$$y - 32 = 20(x - 4)$$

- (c) $y 32 = \frac{5}{2}(x 4)$
- (d) y 32 = 5(x 4)
- (e) y 4 = 5(x 32)

10. (5 pts.) If
$$f(x) = \frac{x^5 + 1}{x^5 - 1}$$
, find $f'(x)$.
(a) $\frac{10x^4}{(x^5 - 1)^2}$
(b) 0
(c) $\frac{-5x^4}{(x^5 - 1)^2}$
(d) $\frac{25x^4}{(x^5 - 1)^2}$

(e)
$$\frac{-10x^4}{(x^5-1)^2}$$

Initials:_____

11. (5 pts.) Suppose f is given by

$$f(x) = \begin{cases} \frac{x^2 - 1}{x^2 + 1} & \text{if } x < 1, \\ 1 & \text{if } x = 1, \\ \frac{1}{x - 1} & \text{if } x > 1. \end{cases}$$

Evaluate $\lim_{x \to 1^{-}} f(x)$.

- (a) 0
- (b) 2

(c)
$$-1$$

- (d) 1
- (e) The one-sided limit does not exist.

Partial Credit

You must show your work on the partial credit problems to receive credit!

12. (15 pts.) Let f(x) and g(x) be functions such that

f(2) = 3, f'(2) = 4, g(2) = 5, g'(2) = 6.Let F(x) = f(x)g(x).

(a) Find F'(2). (Hint: You'll want to use the product rule.)

(b) Find the equation of the tangent line to y = F(x) at x = 2.

13. (15 pts.) Let $f(x) = x^2 + 2x + 3$.

(a.) Compute and simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h}.$$

(b.) Find f'(x) by evaluating the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Note that we're asking you to evaluate a limit, so show us this work. You should make sure your answer coincides with what you know f'(x) to be.

Initials:_____

Initials:_____

14. (15 pts.)

(a) Evaluate

$$\lim_{h \to 0} \frac{\sqrt[3]{1+h} - 1}{h}$$

using what you know about derivatives. Explain your answer - your reasoning is as important as your final answer.

(b) (Unrelated to (a).) For which value of d is the following function continuous at x = 3? Be sure to carefully explain your answer and put your answer in the indicated box.

$$f(x) = \begin{cases} 2x^2 + d & \text{if } x \le 3\\ 4x + 5 & \text{if } x > 3 \end{cases}$$

d =