Name: Solutions

Instructor:________________________

Section number:________________________

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calculus
Fall 2022

Practice Exam 1

September 15, 2022

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. **No books, notes, phones or other aids are permitted.** Be sure to write your name on this title page, and in case pages become detached put your initials at the top of each.

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MC.	______
12.	______
13.	______
14.	______
Tot.	______

Multiple Choice

1. (5 pts.) The expression

$$\frac{(t^2-1)^{-\frac{1}{2}}}{(t^2-3t+2)^{-\frac{3}{2}}}$$

is equal to which of the following expressions?

$$= \frac{(t^2-3t+2)^{3/2}}{(t^2-1)^{1/2}}$$

$$= \frac{[(t-1)(t-2)]^{3/2}}{[(t+1)(t-1)]^{1/2}}$$

$$= \frac{(t-1)^{3/2}(t-2)^{3/2}}{(t+1)^{1/2}(t-1)^{1/2}}$$

$\frac{3}{2}-\frac{1}{2}=1$

$$= \frac{(t-1)(t-2)^{3/2}}{(t+1)^{1/2}}$$

(a) $\dfrac{(t-1)^{\frac{1}{2}}(t-2)^{\frac{3}{2}}}{(t+1)^{\frac{1}{2}}}$

(b) $\dfrac{(t-1)(t-2)^{\frac{3}{2}}}{(t+1)^{\frac{1}{2}}}$

(c) $\dfrac{(t-2)^{\frac{3}{2}}}{(t-1)(t+1)^{\frac{1}{2}}}$

(d) $\dfrac{(t-2)^{\frac{3}{2}}}{(t-1)^{\frac{1}{2}}(t+1)^{\frac{1}{2}}}$

(e) $\dfrac{(t-2)^{\frac{3}{2}}}{(t+1)^{\frac{1}{2}}}$

2. (5 pts.) Read-Til-You-Bleed is a book club. It costs \$60 to be a member, and in addition each member has to pay \$28 for each book they buy. Looks-Like-Books is another book club, which costs \$40 to be a member and each book costs \$32. Frodo wants to join only one of these clubs. How many books would he have to buy so that his total expenses would be equal regardless of which of the two clubs he joins?

Let x = # of books

RTYB cost $= 60 + 28x$

LLB cost $= 40 + 32x$

Set them equal.

$60 + 28x = 40 + 32x$

$20 = 4x$

$x = 5$

(a) 2

(b) 3

(c) 4

(d) 5

(e) 6

3. (5 pts.) Find the values of x that satisfy the inequality

$$\frac{x+2}{2x-1} < 0.$$

(a) $-2 < x < \frac{1}{2}$

(b) $x > -\frac{1}{2}$

(c) $x > -2$

(d) $-\frac{1}{2} < x < 2$

(e) $x < -2$ or $x > \frac{1}{2}$

For the fraction to be negative we need

either (i) $x+2>0$ and $2x-1<0$

or (ii) $x+2<0$ and $2x-1>0$.

(i) means $x>-2$ <u>and</u> $x<\frac{1}{2}$ so $-2<x<\frac{1}{2}$

(ii) means $x<-2$ <u>and</u> $x>\frac{1}{2}$

Since (ii) is impossible, the answer comes from (i).

4. (5 pts.) Find an equation of the line that passes through the point $(1, 2)$ and is parallel to the line whose equation is

$$-6x + 2y + 12 = 0.$$

(a) $y = -3x$

(b) $y = 3x - 5$

(c) $y = 2x + 3$

(d) $y = x - 6$

(e) $y = 3x - 1$

the line can be rewritten as

$2y = 6x - 12$

$y = 3x - 6$

So the slope is 3.

So our line is $y - 2 = 3(x-1)$

$y = 3x - 3 + 2$

$y = 3x - 1$

5. (5 pts.) In the pair of supply and demand equations below, x represents the quantity demanded and p the unit price. Find the equilibrium price. That is, find the value of p where the supply curve and the demand curve intersect.

$$\text{Demand equation: } p(x) = \frac{24}{x+4},$$
$$\text{Supply equation: } p(x) = x + 2.$$

(a) $p = 3$

(b) $p = 6$

(c) $p = 4$

(d) $p = 12$

(e) $p = 2\sqrt{3}$

$$\frac{24}{x+4} = x+2$$

$$24 = (x+2)(x+4)$$
$$= x^2 + 6x + 8$$

So $x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0 \implies x = 2$

then $p(2) = \frac{24}{2+4} = 4$

Note also the supply equation gives $p(2) = 2+2 = 4$

6. (5 pts.) Evaluate the following limit:

$$\lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h} \quad \frac{(\sqrt{9+h} + 3)}{(\sqrt{9+h} + 3)}$$

(a) $\frac{1}{6}$

(b) 6

(c) $\frac{1}{3}$

(d) 3

(e) The limit does not exist.

$$= \lim_{h\to 0} \frac{(\cancel{9}+h) - \cancel{9}}{h(\sqrt{9+h} + 3)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{9+h} + 3} = \frac{1}{3+3} = \frac{1}{6}$$

7. (5 pts.) Evaluate the limit

$$\lim_{x\to\infty} \frac{3x^2}{2x^2+5x-7}. \qquad \frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\infty} \frac{3}{2+5/x-7/x^2} = \frac{3}{2}$$

(a) 0

(b) $\frac{2}{3}$

(c) -7

(d) $\frac{3}{2}$

(e) The limit does not exist

8. (5 pts.)
Let $f(x)$ be a continuous function and assume that we have the following values for $f(x)$:

x	0	1	2	3	4	5	6	7	8
$f(x)$	3	5	-4	-2	1	-2	1	5	3

(For example, this gives $f(2) = -4$.) On which of the following interval(s) is $f(x)$ guaranteed to take the value 2 at some point? (Warning: we asked for the value 2, not 0.)

(a) $[1,2]$, $[6,7]$

(b) $[1,2]$ only

(c) $[1,2]$, $[3,4]$, $[4,5]$, $[5,6]$

(d) $[0,1]$, $[1,2]$, $[2,3]$, $[3,4]$, $[4,5]$, $[5,6]$, $[6,7]$, $[7,8]$

(e) There is no interval where $f(x)$ is guaranteed to take the value 2.

We use the Intermediate Value Theorem. We get

$[1,2]$ since 2 is between 5 and -4

$[6,7]$ since 2 is between 1 and 5

9. (5 pts.) Let

$$f(x) = x^{\frac{5}{2}}.$$

Find the equation of the tangent line when $x = 4$.

(a) $y - 32 = 4(x - 20)$

(b) $y - 32 = 20(x - 4)$

(c) $y - 32 = \frac{5}{2}(x - 4)$

(d) $y - 32 = 5(x - 4)$

(e) $y - 4 = 5(x - 32)$

$f'(x) = \frac{5}{2} x^{3/2}$

$f'(4) = \frac{5}{2}(4)^{3/2}$

$= \frac{5}{2}\left[(4)^{1/2}\right]^3$

$= \frac{5}{2} \cdot 2^3 = 20$ = slope at $x = 4$

Also $f(4) = 4^{5/2} = 32$

So the equation is $y - 32 = 20(x-4)$

10. (5 pts.) If $f(x) = \dfrac{x^5+1}{x^5-1}$, find $f'(x)$.

(a) $\dfrac{10x^4}{(x^5-1)^2}$

(b) 0

(c) $\dfrac{-5x^4}{(x^5-1)^2}$

(d) $\dfrac{25x^4}{(x^5-1)^2}$

(e) $\dfrac{-10x^4}{(x^5-1)^2}$

$f'(x) = \dfrac{(x^5-1)(5x^4) - (x^5+1)(5x^4)}{(x^5-1)^2}$

$= \dfrac{5x^9 - 5x^4 - 5x^9 - 5x^4}{(x^5-1)^2}$

$= \dfrac{-10x^4}{(x^5-1)^2}$

11. (5 pts.) Suppose f is given by

$$f(x) = \begin{cases} \dfrac{x^2-1}{x^2+1} & \text{if } x < 1, \\ 1 & \text{if } x = 1, \\ \dfrac{1}{x-1} & \text{if } x > 1. \end{cases}$$

$\leftarrow$ since its $\lim_{x \to 1^-}$, we only care about this line

Evaluate $\lim_{x \to 1^-} f(x)$.

(a) 0

(b) 2

(c) -1

(d) 1

(e) The one-sided limit does not exist.

as x approaches 1 <u>from the left</u>, f(x) approaches $\frac{0}{2} = 0$

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

12. (15 pts.) Let $f(x)$ and $g(x)$ be functions such that

$$f(2) = 3, \qquad f'(2) = 4, \qquad g(2) = 5, \qquad g'(2) = 6.$$

Let $F(x) = f(x)g(x)$.

(a) Find $F'(2)$. *(Hint: You'll want to use the product rule.)*

Using the product rule we get $F'(x) = f(x)g'(x) + f'(x)g(x)$.

So $F'(2) = f(2)g'(2) + f'(2)g(2)$

$= (3)(6) + (4)(5) = 18 + 20 = 38$

this is the slope of the tangent line to $y = F(x)$ at $x = 2$

(b) Find the equation of the tangent line to $y = F(x)$ at $x = 2$.

We know $F(2) = f(2)g(2) = (3)(5) = 15$.

So the tangent line has equation

$y - 15 = 38(x - 2)$

13. (15 pts.) Let $f(x) = x^2 + 2x + 3$.

(a.) Compute and simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h}.$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left[(x+h)^2 + 2(x+h) + 3\right] - \left[x^2+2x+3\right]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h + \cancel{3} - (\cancel{x^2} + \cancel{2x} + \cancel{3})}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$$

(b.) Find $f'(x)$ by evaluating the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Note that we're asking you to evaluate a limit, so show us this work. You should make sure your answer coincides with what you know $f'(x)$ to be.

From (a) as $h \to 0$ the second term goes to 0

so limit $= f'(x) = 2x+2$

14. (15 pts.)

(a) Evaluate

$$\lim_{h\to 0} \frac{\sqrt[3]{1+h}-1}{h}$$

using what you know about derivatives. Explain your answer - your reasoning is as important as your final answer.

If $f(x) = \sqrt[3]{x} = x^{1/3}$, the limit gives $f'(1)$ (since $\sqrt[3]{1} = 1$)

we know $f'(x) = \frac{1}{3}x^{-2/3}$, so this limit must equal $\frac{1}{3}(1)^{-2/3} = \frac{1}{3}$

(b) (Unrelated to (a).) For which value of d is the following function continuous at $x = 3$? Be sure to carefully explain your answer and put your answer in the indicated box.

$$f(x) = \begin{cases} 2x^2 + d & \text{if } x \le 3 \\ 4x + 5 & \text{if } x > 3 \end{cases}$$

$d =$ -1

Using the second line we get $\lim_{x\to 3^+} f(x) = 4(3)+5 = 17$.

To be continuous we need $\lim_{x\to 3^-} f(x) = 17$.

So $2(3)^2 + d = 17$

$18 + d = 17$

$d = -1$

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