

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I
Fall 2022

Name:_____

Instructor:_____

Time MWF class meets:_____

Practice Exam 2

October 11, 2022

This practice exam is in 2 parts on 11 pages and contains 14 problems. It is a little bit longer than the “real” exam will be (it has more partial credit problems). You should be able to finish it in 1.5 hours. **No books, notes, phones or other aids are permitted.**

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You must record here your answers to the multiple choice problems by placing an \times through your answer to each problem.

- | | | | | | |
|----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
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Multiple Choice

1. (5 pts.) Let $f(x)$ be a continuous function and assume that $f(x)$, $f'(x)$, and $f''(x)$ exist for all real numbers. The table below lists where its first and second derivatives are positive and negative.

Interval	$f'(x)$	$f''(x)$
$(-\infty, -6)$	+	-
$(-6, 0)$	+	+
$(0, 6)$	-	-
$(6, 12)$	-	+
$(12, \infty)$	+	+

[For example, this reads $f'(x)$ is negative on $(6, 12)$.] Which of the following is **FALSE**?

- (a) $f(x)$ has an inflection point at $x = 6$
- (b) $f(x)$ has a relative minimum at $x = 6$
- (c) $f(x)$ has an inflection point at $x = -6$
- (d) $f(x)$ has a relative maximum at $x = 0$
- (e) $f(x)$ has a relative minimum at $x = 12$

2. (5 pts.) Let $P(t)$ denote the population of song sparrows in South Bend in the past decade. The population was increasing, but the rate of increase has becoming smaller over time (i.e. the rate was decreasing). Which of the following is **TRUE**?

- (a) $P'(t) > 0$ and $P''(t) < 0$
- (b) $P'(t) < 0$ and $P''(t) > 0$
- (c) $P'(t) > 0$ and $P''(t) > 0$
- (d) $P'(t) < 0$ and $P''(t) < 0$
- (e) The graph of $P(t)$ is concave up.

3. (5 pts.) Find all intervals where $f(x) = \frac{1}{30}x^6 - \frac{1}{12}x^4 - 1000$ is concave up.

- (a) $(-\infty, 0)$ and $(0, \infty)$
- (b) $(-\infty, -1)$, $(0, 1)$ and $(1, \infty)$
- (c) $(-1, 0)$
- (d) $(-\infty, -1)$ and $(1, \infty)$
- (e) $(-1, 1)$

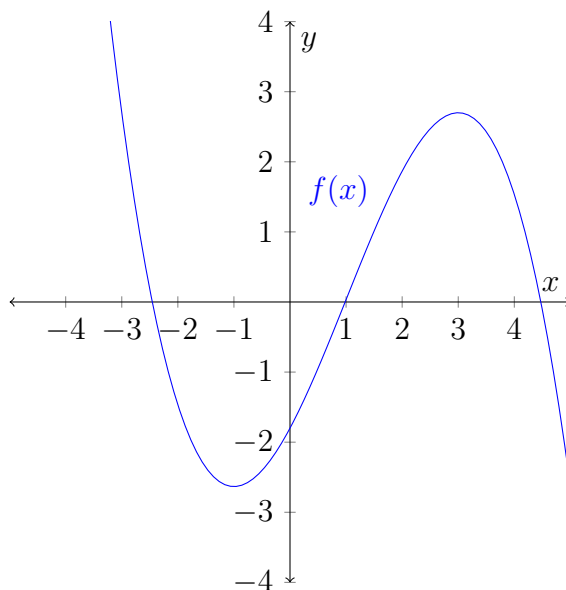
4. (5 pts.) When analyzing a continuous function $h(x)$, Sally finds that $h'(2) = 0$ and $h''(2) = -3$. What can Sally conclude about $h(x)$? (That is, what *must* be true regardless of what $h(x)$ is?)

- (a) $h(x)$ has an inflection point at $x = 2$.
- (b) $h(x)$ has a local maximum at $x = 2$.
- (c) $h(x)$ has a local minimum at $x = 2$.
- (d) $h(x)$ has an absolute minimum at $x = 2$.
- (e) The second derivative test is inconclusive.

5. (5 pts.) What are the inflection points of $Q(x) = x^2 + (x + 2)^{-1}$?

- (a) $Q(x)$ has only one inflection point, at $x = -2$.
- (b) $Q(x)$ has inflection points at $x = -3$ and $x = 0$.
- (c) $Q(x)$ has only one inflection point, at $x = -3$.
- (d) $Q(x)$ has no inflection points.
- (e) $Q(x)$ has inflection points at $x = -3/2$ and $x = 0$.

6. (5 pts.) Consider a function $f(x)$ having the graph below. Exactly **one** of the following statements is **false**. Which is it?



- (a) $f(x)$ is increasing on $(-1, 3)$ and decreasing for $x < -1$ and $x > 3$.
- (b) $f(x)$ has a local minimum at $x = -1$ and a local maximum at $x = 3$.
- (c) $f(x)$ is concave up for $x < 1$ and concave down for $x > 1$.
- (d) $f(x)$ has critical points at $x = -1, x = 1$, and $x = 3$.
- (e) $f(x)$ has an inflection point at $x = 1$.

7. (5 pts.)

Consider the function

$$h(x) = \frac{1}{\sqrt{x-3}}$$

Compute $\lim_{x \rightarrow 3^+} h(x)$ and $\lim_{x \rightarrow \infty} h(x)$.

- (a) $\lim_{x \rightarrow 3^+} h(x) = 0, \lim_{x \rightarrow \infty} h(x) = \infty$
- (b) $\lim_{x \rightarrow 3^+} h(x) = -\infty, \lim_{x \rightarrow \infty} h(x) = 3$
- (c) $\lim_{x \rightarrow 3^+} h(x) = \infty, \lim_{x \rightarrow \infty} h(x) = \infty$
- (d) $\lim_{x \rightarrow 3^+} h(x) = -\infty, \lim_{x \rightarrow \infty} h(x) = 0$
- (e) $\lim_{x \rightarrow 3^+} h(x) = \infty, \lim_{x \rightarrow \infty} h(x) = 0$

8. (5 pts.) Consider the following equation in x and y :

$$x^4 + y^4 = y^2 + 3.$$

Using implicit differentiation, find $\frac{dy}{dx}$ as a function of x and y .

- (a) $\frac{2 - 4x^3}{4y^3 - 2y}$
- (b) $\frac{2y - 4x^3}{4y^3}$
- (c) $\frac{-4x^3}{4y^3 - 2y}$
- (d) $\frac{4x^3 + 4y^3}{2y}$
- (e) $\frac{2y + 2 - 4x^3}{4y^3}$

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

9. (15 pts.) Consider the function $y = f(x) = -x^3 + 3x$.

- (a) Determine the intervals where the graph of $f(x)$ is increasing and where it is decreasing.

- (b) Find the points (x, y) which are relative extrema of $f(x)$. (*Don't forget the y-value.*) Label which are maxima and which are minima.

- (c) Determine the intervals where the graph of $f(x)$ is concave up and where it is concave down.

- (d) Find the points (x, y) that are inflection points of $f(x)$.

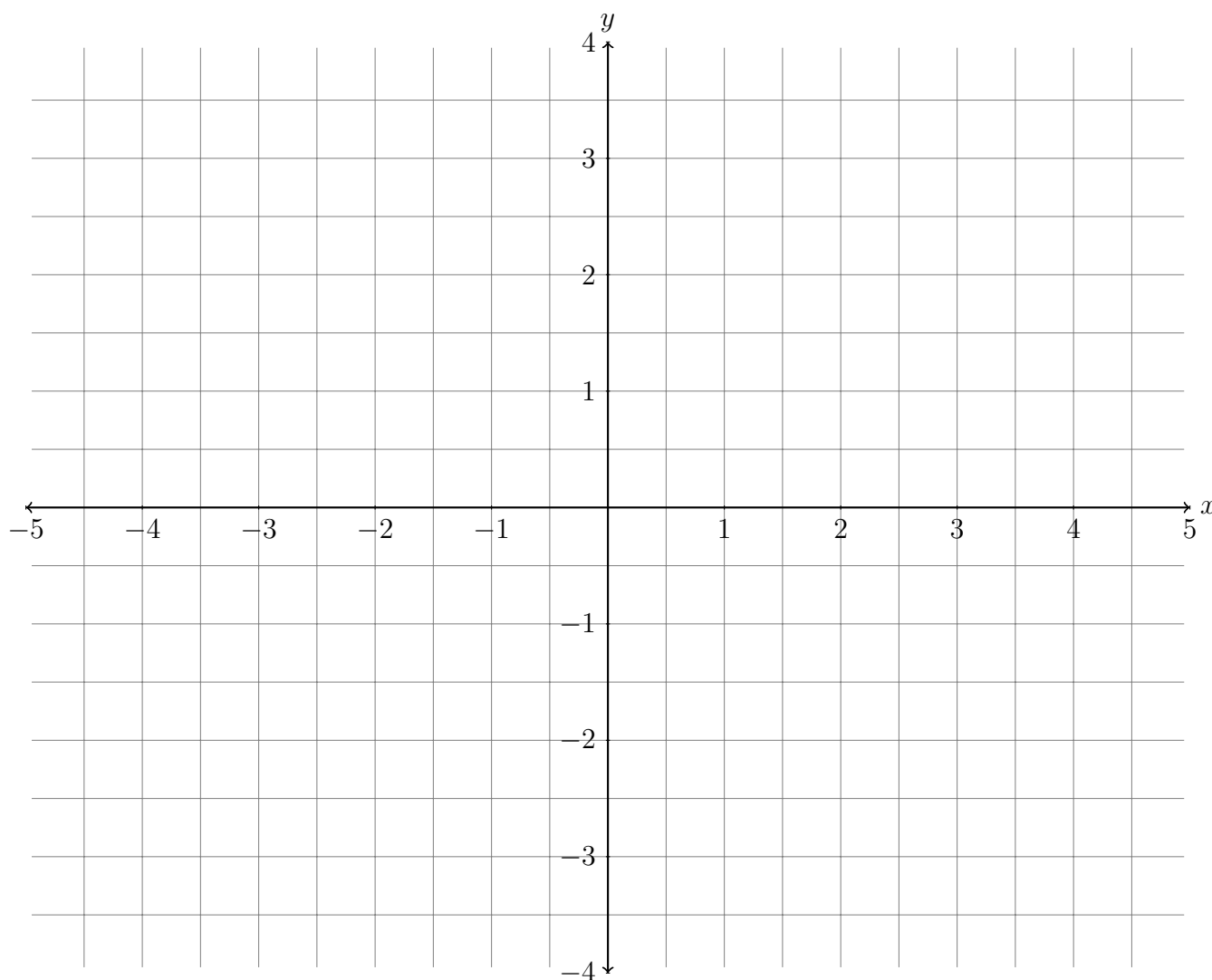
10. (15 pts.) Draw the graph of a continuous function $y = f(x)$ satisfying the following list of properties. Specify the coordinates (x, y) of any relative minima, relative maxima, and inflection points in the blanks below (write NONE if there are none). Draw any asymptotes with a dotted line.

- The domain is all real numbers.
- $f(-2) = 2$, $f(0) = 0$, and $f(2) = -2$.
- $f'(x) < 0$ on $(-2, 2)$.
- $f'(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$.
- $f'(x) = 0$ at $x = -2$ and $x = 2$
- $f''(x) < 0$ on $(-\infty, 0)$
- $f''(x) > 0$ on $(0, \infty)$
- $\lim_{x \rightarrow -\infty} f(x) = -\infty$
- $\lim_{x \rightarrow \infty} f(x) = \infty$

Relative maxima at _____

Relative minima at _____

Inflection point(s) at _____



11. (15 pts.) A bowling ball is dropped from a helicopter into a perfectly calm lake, causing a circular ripple whose radius expands at a rate of 5 feet per second. When the radius reaches 10 feet, at what rate is the area of the circle increasing? (Remember that the area of a circle is $A = \pi r^2$, where r is the radius and A is the area.)

Be sure to explain your work, including a discussion of what is true at all moments and what is true at a certain moment. You might lose points for insufficient explanation even if your final answer is correct.

12. (15 pts.) Consider the function

$$R(x) = \frac{3x^2}{x^2 - 9}$$

(a) Find all the horizontal asymptotes of $R(x)$.

(b) Find all the vertical asymptotes of $R(x)$.

(c) For each vertical asymptote $x = a$ found in part (b), compute $\lim_{x \rightarrow a^+} R(x)$ and $\lim_{x \rightarrow a^-} R(x)$.

- 13.** (15 pts.) In this problem, y is a function of x , and x is a function of t . Specifically,

$$y = x^5$$

$$x = \sqrt{t^3 + 1}$$

- (a) Find $\frac{dy}{dx}$ and $\frac{dx}{dt}$ and put your answer in the appropriate space in the box:

$$\frac{dy}{dx} =$$

$$\frac{dx}{dt} =$$

- (b) Find $\left. \frac{dy}{dt} \right|_{t=2}$ (your answer should be a number).

$$\left. \frac{dy}{dt} \right|_{t=2} =$$

14. (15 pts.) Schwartz & Sons makes solid gold model cars. Let $R(x)$ be their revenue function, where x is the number of model cars produced.

- (a) If the company charges \$500 per car, no matter what x is, what is $R(x)$ and what is the marginal revenue function?

- (b) The executive in charge of setting prices has been sacked, and his replacement now has established a price function

$$p(x) = 200 + \frac{500}{1+x}.$$

What is the new revenue function and what is the new marginal revenue function?

- (c) The company realizes that it is in their interest to make very many of their cars (i.e. x will get very large). What is the limiting value of the **price** they charge per car, using the formula for $p(x)$ in (b)?

- (d) If the cost function is $C(x) = \sqrt{2x+1}$, find the average cost and the marginal average cost functions.

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