Name:________________________________

Instructor:______________________________

Time MWF class meets:________________

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I
Fall 2022

Practice Exam 3 Solutions

November 17, 2022

This exam is in 2 parts on 9 pages and contains 12 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. **No books, notes, phones or other aids other than calculators are permitted.** Be sure to write your name on this title page, and in case pages become detached put your initials at the top of each.

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Multiple Choice

1. (5 pts.) Find the absolute minimum (y-value) of $f(x) = x^3 - 12x + 13$ on the interval $[-3, 1]$.

(a) 2 (b) -3

(c) 0 (d) 29

(e) There is no absolute minimum value on $[-3, 1]$.

Solution:
$f'(x) = 3x^2 - 12$, which is defined for all x. Setting $f'(x) = 0$ we get $3x^2 = 12$, so $x = \pm 2$. Of these, only -2 is in the interval $[-3, 1]$. We compute $f(-3) = 22$, $f(-2) = 29$, $f(1) = 2$ so the latter is the minimum.

2. (5 pts.) Let $f(x) = \ln(e^{3x})$. Find the values of x where the tangent to the graph of $f(x)$ is horizontal.

(a) Only at $x = 1$.

(b) Only at $x = e$.

(c) Only at $x = 0$.

(d) All numbers x.

(e) No numbers x.

Solution: We have $f(x) = \ln(e^{3x}) = 3x$, so $f'(x) = 3$. The tangent is horizontal when $f'(x) = 0$. No number x satisfies this equation.

3. (5 pts.) Evaluate the limit:

$$\lim_{t\to\infty}(72 - 27e^{-72.27t})$$

(a) 45

(b) 72

(c) 71

(d) ∞

(e) $-\infty$

Solution: The key observation is that when t approaches infinity, $-72.27t$ approaches negative infinity. Therefore, $e^{-72.27t}$ approaches 0. Further applying the rules of limits, we have:

$$\lim_{t\to\infty}(72 - 27e^{-72.27t}) = \lim_{t\to\infty}(72) - \lim_{t\to\infty}(27e^{-72.27t}) = 72 - 27\lim_{t\to\infty}(e^{-72.27t}) = 72 - 27\cdot 0 = 72.$$

4. (5 pts.) Solve the equation $\ln(x+2) = \ln(x^2+2x)$. (Hint: be careful about the domain of the natural log.)

(a) $x = 1$

(b) $x = 0$

(c) $x = 1$ and $x = e$

(d) $x = -2$

(e) There are no solutions.

Solution: We have that $e^{\ln(x+2)} = e^{\ln(x^2+2x)}$, so $x+2 = x(x+2)$ or $(x+2)(x-1) = 0$. Since $x = -2$ is not in the domain of $\ln(x+2)$, the only solution is $x = 1$.

5. (5 pts.) Find the derivative of the function $f(x) = x\ln(x^2)$.

(a) 2

(b) $\ln(x^2) + \frac{1}{x}$

(c) $\frac{2}{x}$

(d) $2\ln(x) + 2$

(e) $2\ln(x) + 2x$

Solution: Using the product rule and then the chain rule, we get:

$$f'(x) = (x)' \cdot \ln(x^2) + x \cdot (\ln(x^2))' = \ln(x^2) + x\frac{2x}{x^2} = \ln(x^2) + 2 = 2\ln(x) + 2.$$

6. (5 pts.) Suppose that an investment grows at a rate of 4% compounded **twice a year**. If the initial investment is $P_0 = 900$, which of the following expressions gives the accumulated amount of the investment after 4 years?

(a) $900 \cdot (1.04)^4$

(b) $900 \cdot (1.04)^8$

(c) $900 \cdot (1.02)^4$

(d) $900 \cdot (1.02)^8$

(e) $900 \cdot (1.02)^2$

Solution: According the formula $P(t) = P_0(1 + \frac{r}{n})^{nt}$, we have $P(4) = 900 \cdot (1.02)^8$.

7. (5 pts.) If $h(t) = Ae^{kt}$ and if $h(0) = 3$ and $h(1) = 6$, which of the functions below is equal to $h(t)$?

(a) $h(t) = 2 \cdot 3^t$

(b) $h(t) = 2 \cdot e^t$

(c) $h(t) = 3 \cdot 2^t$

(d) $h(t) = 3 \cdot e^t$

(e) The function can not be determined from the given information.

Solution: We have $3 = h(0) = Ae^{0 \cdot t} = A$, so $A = 3$. In addition, $6 = h(1) = Ae^{1 \cdot k} = 3e^k$, so $e^k = 2$ and $Ae^{kt} = A(e^k)^t = 3 \cdot 2^t$.

8. (5 pts.) For $x > 0$, which of the following functions $F(x)$ is an **antiderivative** of

$$f(x) = \ln(x)?$$

(In other words, for which of the following is it true that $F'(x) = f(x)$?)

(a) $F(x) = x\ln(x) - x$

(b) $F(x) = \dfrac{1}{x}$

(c) $F(x) = \dfrac{\ln(x)}{x}$

(d) $F(x) = \ln(x) + C$ (C is a constant)

(e) $F(x) = C - \ln(x^2)$ (C is a constant).

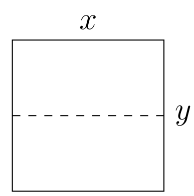
Solution: Taking the derivatives of the proposed functions, we find that

$$(x\ln(x) - x)' = 1 \cdot \ln(x) + x \cdot (\ln(x))' - 1 = \ln(x) + \frac{x}{x} - 1 = \ln(x).$$

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

9. (15 pts.) A peafowl rancher wants to build a rectangular pen which encloses 500 ft^2. In order to separate the peacocks from the peahens, he divides the pen into two equal sections with a barrier that is parallel to one of the sides. The fencing around the perimeter of the pen costs \$1 per foot, and the barrier costs \$8 per foot. Let x and y denote the side lengths of the pen, and assume the barrier is parallel to the side of length x.



(a) Write an expression for y in terms of x.

Solution: The area of the pen is given by

$$\text{Area} = xy = 500$$

Solving for y gives $y = \frac{500}{x}$.

(b) Find the total cost function for building the pen $C(x)$ as a function of x only.

Solution:

The cost is given by

$$\text{Total cost} = \text{Cost of perimeter} + \text{Cost of barrier}$$
$$= (2x + 2y) + (8x) = 10x + 2y$$

Replacing y with our expression from part (a) gives

$$C(x) = 10x + \frac{1000}{x}$$

(c) Find the critical numbers in the domain of $C(x)$.

Solution: Note that x can take any positive value, so $C(x)$ has domain $(0, \infty)$.

$$C'(x) = 10 - \frac{1000}{x^2}$$

$C'(x)$ doesn't exist at $x = 0$, and $C'(x) = 0$ when

$$0 = 10x^2 - 1000 = 10(x^2 - 100)$$

i.e. $x = \pm 10$. The only critical number in the domain is $x = 10$.

(d) Find the dimensions (x and y) that achieve the minimum cost. (Be sure to justify that this is the absolute minimum)

Solution:

$C''(x) = \frac{2000}{x^3}$, which is always positive on our domain. In particular, $C''(10) > 0$ is a relative min, and since the function is always concave up, it is an absolute min. When $x = 10$, $y = \frac{500}{10} = 50$.

10. (15 pts.) A rabbit population grows according to the equation

$$P(t) = A_0 e^{kt},$$

where time is measured in weeks. Initially, there are 12 rabbits. After 2 weeks, the population has grown to 15 rabbits.

(a) Find A_0.
Solution: We know that $P(0) = A_0 e^{0 \cdot t} = A_0$. In addition, $P(0) = 12$. Therefore, $A_0 = 12$.

(b) Find the value of k.

Solution: Using part (a), we have that $P(2) = 12e^{2k}$. Since $P(2) = 15$, we have $e^{2k} = \frac{15}{12}$ so $k = \frac{1}{2}\ln(\frac{5}{4}) \approx 0.112$.

(c) Write down a formula for $P(t)$.
Solution: Using parts (a) and (b), we find that $P(t) = 12(\frac{5}{4})^{\frac{1}{2}t} \approx 12e^{0.112t}$.

(d) How long does it take, in weeks, for the population of rabbits to reach 30?

Solution: We want to solve for t the equation $12(\frac{5}{4})^{\frac{1}{2}t} = 30$ or $(\frac{5}{4})^{\frac{1}{2}t} = \frac{5}{2}$. Taking the natural log of both sides, we have $\frac{1}{2}t \cdot \ln(\frac{5}{4}) = \ln(\frac{5}{2})$ or $t = \dfrac{2\ln(\frac{5}{2})}{\ln(\frac{5}{4})} \approx 8.21$ weeks.

11. (15 pts.)

(a) Let $f(x) = (\ln(x))^7$. Find $f'(x)$.

Solution: $f'(x) = 7(\ln(x))^6(\ln(x))' = \dfrac{7(\ln(x))^6}{x}$.

(b) Let $f(x) = x\sqrt{e^x}$. Find $f'(x)$.

Solution: We can rewrite the function as $f(x) = xe^{0.5x}$. We now find its derivative: $f'(x) = e^{0.5x} + 0.5xe^{0.5x}$.

(c) $\displaystyle\int \left(\frac{1}{x} - \sqrt{x} + 1\right) dx = \int \frac{1}{x}dx - \int x^{\frac{1}{2}}dx + \int 1dx = \ln|x| - \frac{2}{3}x^{\frac{3}{2}} + x + C$

(d) $\displaystyle\int 8dz = 8z + C$

12. (15 pts.)
Let $f(x)$ be a function with the property that

$$f'(x) = \frac{1}{\sqrt{x}}.$$

(a) Find a general formula for $f(x)$. (Your answer should involve an unknown constant C.)

Solution: We have that $f(x) = \int \frac{1}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} + C$ for some constant C.

(b) If $f(4) = 2$, find $f(x)$.

Solution: From (a), $f(4) = 2 \cdot \sqrt{4} + C = 2 \cdot 2 + C = 4 + C$. Since $f(4) = 2$, we find that $C = -2$ and $f(x) = 2x^{0.5} - 2$.

(c) Evaluate $f(9)$.

Solution: From (a) and (b) we know that $f(9) = 2\sqrt{9} - 2 = 2 \cdot 3 - 2 = 4$.

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