Math 10250 Elements of Calculus
Practice Final Exam
December 13, 2022

Name: ______________________________

- This exam is on 15 pages and contains 25 problems worth a total of 150 points. You have 2 hours to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page (where it says "Name") in case pages become detached.

  You must record on this page your answers to all of the problems.

  Place an $\times$ through your answer to each problem.

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Multiple Choice

1. (6 pts.) The derivative of $f(x) = e^{\frac{1}{4x}}$ is:

(a) $-\dfrac{e^{\frac{1}{4x}}}{4}$

(b) $\dfrac{e^{\frac{1}{4x}}}{x}$

(c) $\dfrac{e^{\frac{1}{4x}}}{4x^2}$

(d) $\dfrac{e^{\frac{1}{4x}}}{x^2}$

(e) $-\dfrac{e^{\frac{1}{4x}}}{4x^2}$

$$f'(x) = e^{1/4x}\left(\frac{1}{4x}\right)' = e^{\frac{1}{4x}}\left(\frac{1}{4}x^{-1}\right)'$$

$$= \frac{1}{4}e^{\frac{1}{4x}}\left(-x^{-2}\right)$$

$$= -\frac{e^{1/4x}}{4x^2}$$

2. (6 pts.) Find the x-coordinate of the point on the graph of $y = x^3 - 3x^2 + 12$ where the tangent line is parallel to the line $y = -3x - 9$.

(a) $x = 1$

(b) $x = 0$

(c) $x = \frac{1}{2}$

(d) $x = -2$

(e) $x = \frac{2}{3}$

the line has slope -3 so we want to know where $\frac{dy}{dx} = -3$

$\frac{dy}{dx} = 3x^2 - 6x$ so we set

$$3x^2 - 6x = -3$$

$$3x^2 - 6x + 3 = 0$$

$$3(x^2 - 2x + 1) = 0$$

$$3(x-1) = 0$$

$$x = 1$$

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3. (6 pts.) Using implicit differentiation, find the slope of the tangent line to the curve

$$\ln(x^2 + y) + y = 0$$

at the point $(1, 0)$. [Hint: in other words, find $\frac{dy}{dx}$.]

Differentiate wrt x

$$\frac{1}{x^2+y}\left(2x + \frac{dy}{dx}\right) + \frac{dy}{dx} = 0$$

$$\frac{2x}{x^2+y} + \frac{1}{x^2+y}\frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[\frac{1}{x^2+y} + 1\right] = -\frac{2x}{x^2+y}$$

Plug in $x=1, y=0$

$$\frac{dy}{dx}[1+1] = -\frac{2}{1} = -2, \quad \frac{dy}{dx} = \frac{-2}{2} = -1$$

(a) 2

(b) -1

(c) $2e$

(d) $\frac{1}{e}$

(e) 0

4. (6 pts.) If $f(x) = 5\sqrt{x}$ and $g(x) = e^x + 1$, then $(f \circ g)(x)$ is given by

$$(f \circ g)(x) = f(g(x)) = 5\sqrt{e^x + 1}$$

(a) $e^{5\sqrt{x}} + 1$

(b) $e^{\sqrt{5x}} + 1$

(c) $e^{\sqrt{5x+1}}$

(d) $5\sqrt{e^x} + 1$

(e) $5\sqrt{e^x + 1}$

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5. (6 pts.) Let

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1; \\ c & \text{if } x = 1. \end{cases}$$

Which choice of c will make f continuous at $x = 1$?

$$\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right) = \lim_{x\to 1}\frac{(x+1)(x-1)}{x-1}$$

$$= \lim_{x\to 1}(x+1) = 2$$

(a) $c = 1$

(b) $c = 2$

(c) $c = -1$

(d) $c = 0$

(e) No value of c can make the function continuous.

6. (6 pts.) A molasses factory experiences a major leak, which forms a puddle in the shape of a perfect circle. If the radius of the puddle is growing at a rate of 3ft per minute, at what rate in the area of the puddle increasing when the radius is 10 ft? (Hint: the area of a circle of radius r is given by the formula πr^2.)

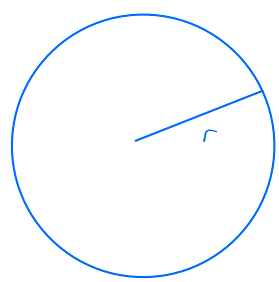
(a) 30π ft^2 per minute

(b) 90π ft^2 per minute

(c) 300π ft^2 per minute

(d) 60π ft^2 per minute

(e) The area of the puddle stays constant because of the density.



$A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi r(3) = 6\pi r$$

Plug in $r = 10$

$$\frac{dA}{dt} = 60\pi$$

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7. (6 pts.) An investor's initial investment of \$10,000 grew to \$15,000 in 10 years. Which expression gives the rate of interest on the investment, assuming the interest was compounded **continuously**?

(a) $10\ln(1.5)$

(b) $\frac{2}{3}e^{10}$

(c) $\frac{1}{10}\ln(1.5)$

(d) $(1.1)^{\frac{1}{10}} - 1$

(e) 0.05

$A(t) = 10{,}000\, e^{rt}$

$15{,}000 = 10{,}000\, e^{10r}$

$\frac{3}{2} = e^{10r}$

$\ln\left(\frac{3}{2}\right) = 10r$

$r = \frac{1}{10}\ln(3/2)$

8. (6 pts.) A savings account with an interest rate of 2% compounded **twice per year** reaches an accumulated amount of \$64,000 in 8 years. Which of the following expressions represents the initial (or principal) investment?

(a) $64,000 \cdot (1.01)^{8}$

(b) $64,000 \cdot (1.02)^{-8}$

(c) $64,000 \cdot (1.02)^{-16}$

(d) $64,000 \cdot (1.01)^{-16}$

(e) $64,000 \cdot (1.02)^{8}$

$A(t) = A_0\left(1 + \frac{.02}{2}\right)^{2t}$

$64{,}000 = A_0(1 + .01)^{16}$

$A_0 = 64{,}000\,(1.01)^{-16}$

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$$5^{x^2+1} = \frac{1}{(5^2)^x} = \frac{1}{5^{2x}} = 5^{-2x}$$

9. (6 pts.) Solve for x in the equation $5^{x^2+1} = \dfrac{1}{25^x}$

(a) $x = -1$

(b) $x = 1$ and $x = -1$

(c) $x = 0$ and $x = 1$

(d) $x = 2$

(e) There are no solutions.

So $x^2 + 1 = -2x$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1$

10. (6 pts.) Consider the natural logarithmic function $f(x) = \ln(x)$. Which of the following is **FALSE**?

(a) $\ln(x)$ passes through the point $(1, 0)$. true

(b) $\ln(x)$ is always positive. false

(c) $\ln(x)$ has a vertical asymptote. true

(d) $\ln(x)$ has no horizontal asymptotes. true

(e) The domain of $\ln(x)$ is all $x > 0$. true

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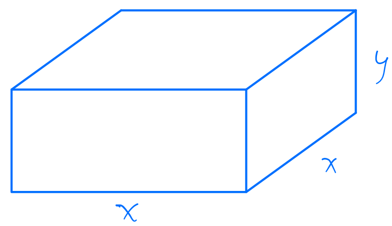
11. (6 pts.) A rectangular pencil container with square base and open top will have a volume of 40 in^3. The cost for the base is \$ 0.40 per square inch, while the cost for the sides is \$0.15 per square inch. Suppose the container was constructed to minimize cost. What is the side length of its base?



(a) 3

(b) $\sqrt{24}$

(c) 4

(d) No side length minimizes the cost.

(e) $\sqrt[3]{30}$

$x^2 y = 40 \rightsquigarrow y = \frac{40}{x^2}$

$C(x) = 0.15(4)\left(x\left(\frac{40}{x^2}\right)\right) + 0.40(x^2)$

$= \frac{24}{x} + .4x^2$

$C' = -24x^{-2} + 0.8x$

Set $C' = 0$

$0.8x = \frac{24}{x^2}$

$x^3 = 30 \quad x = \sqrt[3]{30}$

Check min:

$-$ $\sqrt{30}$ $+$ C'

so this is a min

12. (6 pts.) The demand function for Bunny Beach Balls is given by $p(x) = \sqrt{900 - x}$, where p is the price in dollars and x is the number of beach balls made. Because of supply chain issues, at most 660 beach balls can be made per week. How many beach balls should the company make to maximize its **revenue**?

(a) 660

(b) 0

(c) 600

(d) 500

(e) A maximum revenue cannot be achieved.

$R = \text{Revenue} = px$

$= x(900-x)^{1/2}$

$R' = x\left(\frac{1}{2}\right)(900-x)^{-1/2}(-1) + (900-x)^{1/2}$

$= \sqrt{900-x} - \frac{x}{2\sqrt{900-x}}$

Set $R' = 0$

$\sqrt{900-x} = \frac{x}{2\sqrt{900-x}}$

$2(900-x) = x$

$1800 - 2x = x$

$1800 = 3x$

$x = 600$

don't forget to check it's a min and not a max.

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13. (6 pts.) On which interval is $f(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 9x^2 + 5$ concave down?

(a) $(2, \infty)$

(b) $(-\infty, 2)$

(c) $(-3, 2)$

(d) $(-\infty, -3)$

(e) $(5, \infty)$

f'': $+$ $\;-3\;$ $-$ $\;2\;$ $+$

$f' = x^3 + \frac{3}{2}x^2 - 18x$

$f'' = 3x^2 + 3x - 18$

$= 3(x^2 + x - 6)$

$= 3(x+3)(x-2)$

14. (6 pts.) How many critical points does $g(x) = \dfrac{x}{x^2+4}$ have?

(a) 2

(b) 1

(c) 0

(d) 3

(e) 4

$g' = \dfrac{(x^2+4)(1) - x(2x)}{(x^2+4)^2}$

$= \dfrac{4 - x^2}{(x^2+4)^2}$

$= \dfrac{(2+x)(2-x)}{(x^2+4)^2}$

set $g' = 0$. The roots are $x = 2, -2$

Also the denominator is never 0.

So there are 2 critical points

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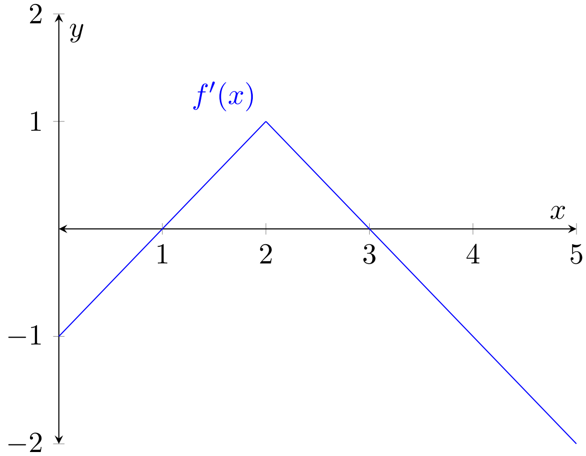
15. (6 pts.) The **derivative** $f'(x)$ of a function is graphed below.



Where does the **original function** $f(x)$ have a relative maximum?

(a) $x = 1$

(b) $x = 2$

(c) $x = 4$

(d) $x = 3$

(e) $x = 5$

f' is negative on $(0,1)$ and on $(3,\infty)$

f' is positive on $(1,3)$

So f' changes from positive to negative at $x=3$

16. (6 pts.) Let

$$f(x) = (e^{2x} + 1)(\ln(x)).$$

Find $f'(x)$.

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(a) $\dfrac{e^{2x}+1}{x} + 2(\ln(x))(e^{2x}+1)$

(b) $\dfrac{2e^{2x}}{x}$

(c) $\dfrac{e^{2x}+1}{x} + 2(\ln(x))e^{2x}$

(d) $\dfrac{e^{2x}+1}{x} + (\ln(x))e^{2x}$

(e) $\dfrac{e^{2x}+1}{x}$

$$f' = (e^{2x}+1)\left(\frac{1}{x}\right) + (\ln x)(e^{2x})(2)$$

$$= \frac{e^{2x}+1}{x} + 2(\ln x)(e^{2x})$$

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17. (6 pts.) Find the equation of the tangent line to the graph of $y = e^{(x^2+1)}$ at the point on the graph where $x = 2$.

if $x=2$ then $y = e^5$

$\frac{dy}{dx} = e^{(x^2+1)}(2x)$

slope $= \left.\frac{dy}{dx}\right|_{x=2} = e^5(4)$

(a) $y - e^5 = e^5(x-2)$

(b) $y - e^2 = 4e^2(x-2)$

(c) $y - e^5 = 2e^5(x-2)$

(d) $y - e^2 = e^2(x-2)$

(e) $y - e^5 = 4e^5(x-2)$

so $y - e^5 = 4e^5(x-2)$

18. (6 pts.) Use logarithmic differentiation to find $\frac{dy}{dx}$ if

$$y = e^x(2x+1)^3(4x+2)^5.$$

(a) $\frac{dy}{dx} = \left(1 + \frac{3}{2x+1} + \frac{5}{4x+2}\right) \cdot e^x(2x+1)^3(4x+2)^5$

(b) $\frac{dy}{dx} = \left(1 + \frac{6}{2x+1} + \frac{20}{4x+2}\right) \cdot e^x(2x+1)^3(4x+2)^5$

(c) $\frac{dy}{dx} = \left(x + \frac{3}{2x+1} + \frac{5}{4x+2}\right) \cdot e^x(2x+1)^3(4x+2)^5$

(d) $\frac{dy}{dx} = \left(1 + \frac{6}{2x+1} + \frac{20}{4x+2}\right)$

(e) $\frac{dy}{dx} = \left(1 + \frac{3}{2x+1} + \frac{5}{4x+2}\right)$

Take ln on both sides:

$\ln y = x + 3\ln(2x+1) + 5\ln(4x+2)$

Differentiate:

$\frac{1}{y}\frac{dy}{dx} = 1 + \frac{3}{2x+1}(2) + \frac{5}{4x+2}(4)$

So

$\frac{dy}{dx} = \left(1 + \frac{6}{2x+1} + \frac{20}{4x+2}\right)\left(e^x(2x+1)^3(4x+2)^5\right)$

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19. (6 pts.) The height of the members of a certain species of animal is approximated by the function

$$h(t) = 5(1 - 4e^{-3t})$$

where t is measured in years and $h(t)$ is measured in feet. The animals live a **very** long time. According to this model, what is the approximate height of a very old member of this species (i.e. what is the height as his age approaches infinity)?

(a) 5 feet

(b) 1 foot

(c) 4 feet

(d) 20 feet

(e) 0 feet

$\lim_{t \to \infty} h(t) = \lim_{t \to \infty} 5\left(1 - \frac{4}{e^{3t}}\right)$

As $t \to \infty$, the term $\frac{4}{e^{3t}}$ goes to 0

So the limit is 5.

20. (6 pts.) Solve the following initial-value problem:

$$f'(x) = 2 + \frac{1}{x}, \qquad f(1) = 6.$$

(a) $f(x) = \ln(x) + 6$

(b) $f(x) = 2\ln(x) + 6$

(c) $f(x) = 2x + \ln(x) + 4$

(d) $f(x) = 2x + \ln(x) + 6$

(e) $f(x) = 2x + \ln(x)$

$f(x) = \int \left(2 + \frac{1}{x}\right) dx$

$= 2x + \ln x + C$

$f(1) = 6$ means

$2 + \ln(1) + C = 6$

$2 + C = 6$

$C = 4$

$f(x) = 2x + \ln x + 4$

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21. (6 pts.) Evaluate the following definite integral:

$$\int_1^3 (x^2+1)dx.$$

$\left[\frac{1}{3}x^3 + x\right]_1^3 = \left(9+3\right) - \left(\frac{1}{3}+1\right) = 12 - \frac{4}{3} = \frac{32}{3}$

(a) 12

(b) 11

(c) 28

(d) $\frac{32}{3}$

(e) 8

22. (6 pts.) Evaluate the following definite integral:

$$\int_0^2 3^x dx.$$

[Hint: Remember that $3^x = e^{x\ln(3)}$ so use a u-substitution.]

(a) $\frac{9}{\ln(3)}$

(b) $\frac{7}{\ln(3)}$

(c) $\frac{11}{\ln(3)}$

(d) $\frac{8}{\ln(3)}$

(e) $\frac{10}{\ln(3)}$

$\int_0^2 \left(e^{\ln 3}\right)^x dx = \int_0^2 e^{x\ln 3} dx$

$u = x\ln 3$

$du = (\ln 3)dx$

$\frac{1}{\ln 3} du = dx$

when $x=0$, $u=0$

when $x=2$, $u=2\ln 3$

$= \frac{1}{\ln 3}\int_0^{2\ln 3} e^u du = \frac{1}{\ln 3}\left(e^{2\ln 3} - e^0\right)$

$= \frac{1}{\ln 3}(9-1)$

$= \frac{8}{\ln 3}$

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23. (6 pts.) Find the indefinite integral:

$$\int \frac{(\ln(x))^2}{x} dx \quad = \int u^2 du = \frac{1}{3}u^3 + C$$

$u = \ln x$ $\qquad = \frac{1}{3}(\ln x)^3 + C$

$du = \frac{1}{x} dx$

(a) $\frac{1}{3}(\ln(x))^3 + C$

(b) $\frac{\frac{1}{3}(\ln(x))^3}{\frac{1}{2}x^2} + C$

(c) $2\ln(x) + C$

(d) $\frac{1}{3}\ln\left(\frac{1}{2}x^2\right) + C$

(e) $\frac{2\ln(x)}{x^2} + C$

24. (6 pts.) Let $f(x) = x^2 + 1$. Estimate $\int_1^3 f(x)dx$ using a Riemann sum with $n = 4$ subintervals and using the left endpoint of the subintervals for the height of the rectangles.

(a) $\left[1 + \frac{3}{2} + 2 + \frac{5}{2}\right] \cdot \left(\frac{1}{2}\right)$

(b) $\left[1 + \frac{9}{4} + 4 + \frac{25}{4}\right] \cdot \left(\frac{1}{2}\right)$

(c) $\left[2 + \frac{5}{2} + 3 + \frac{7}{2}\right] \cdot \left(\frac{1}{2}\right)$

(d) $[2 + 5] \cdot (1)$

(e) $\left[2 + \frac{13}{4} + 5 + \frac{29}{4}\right] \cdot \left(\frac{1}{2}\right)$

$f(1) = 2$

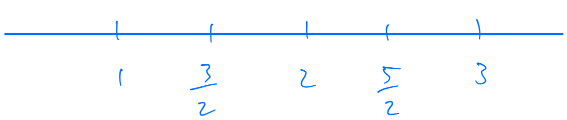
$f\left(\frac{3}{2}\right) = \frac{9}{4} + 1 = \frac{13}{4}$

$f(2) = 5$

$f\left(\frac{5}{2}\right) = \frac{25}{4} + 1 = \frac{29}{4}$

$\Delta x = \frac{1}{2}$

$\left[2 + \frac{13}{4} + 5 + \frac{29}{4}\right]\left(\frac{1}{2}\right)$



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25. (6 pts.) Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[1, 9]$.

(a) $\frac{13}{4}$

(b) $\frac{13}{6}$

(c) $\frac{52}{3}$

(d) $\frac{1}{2}$

(e) $\frac{1}{4}$

$$\frac{1}{9-1}\int_1^9 \sqrt{x}\,dx = \frac{1}{8}\left[\frac{2}{3}x^{3/2}\right]_1^9$$

$$= \left(\frac{1}{8}\right)\left(\frac{2}{3}\right)\left[9^{3/2} - 1^{3/2}\right]$$

$$= \frac{1}{12}\left[27-1\right] = \frac{26}{12} = \frac{13}{6}$$

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