Math 10250 Elements of Calculus
Practice Final Exam 2
December 13, 2022

Name: _____

- This exam is on 14 pages and contains 25 problems worth a total of 150 points. You have 2 hours to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

  You must record on this page your answers to all of the problems.

  Place an × through your answer to each problem.

Honor Pledge: _____

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
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13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						
15.	(a)	(b)	(c)	(d)	(e)						
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18.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1. (5 pts.) Find the equation of the tangent line to the curve $y = x^3$ at the point where $x = 2$.

(a) $y = 12x - 24$ (b) $y = 12x - 8$ (c) $y = 12x - 16$

(d) $y = 8x - 12$ (e) $y = 8x - 16$

2. (5 pts.) Find the slope of the tangent line to the curve

$$y^2 + xe^y = 1$$

at the point $(1, 0)$.

(a) $\frac{1}{2}$ (b) $-e^y/x$ (c) 1

(d) -1 (e) 0

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3. (5 pts.) A snowball in the shape of a perfect sphere is melting in such a way that its volume is decreasing at a rate of 36π in^3 per minute. How fast is the radius decreasing when the radius is 3 in? Remember that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

(a) 1 in per min (b) 2 in per min (c) 3 in per min

(d) 2π in per min (e) 4 in per min

4. (5 pts.) A company that sells canned tuna needs to design a cylindrical can whose volume is 16π in^3. The top and bottom of the can are made from aluminum, which costs \$.01 per in^2. However, for structural reasons the side of the can needs to be made from a different metal, which costs \$.08 per in^2. What are the radius and height (in inches) of the can which will minimize the total cost of the material needed to construct a can? [Hint: for a cylinder of radius r and height h, the volume is $\pi r^2 h$, the area of the side is $2\pi rh$ and the area of the top and bottom combined is $2\pi r^2$.]

(a) $r = 2,\ h = 4$ (b) $r = 1,\ h = 16$ (c) $r = 8,\ h = 1/4$

(d) $r = \sqrt{2},\ h = 8$ (e) $r = 4,\ h = 1$

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5. (5 pts.) Solve the initial value problem

$$y' = \frac{2x+3}{x^2+3x+2},$$

$$y(0) = 1 + \ln 2.$$

Assume that $x > 0$.

(a) $y = \ln(x^2+3x+2) + 2 + \ln 2$

(b) $y = \ln(x^2+3x+2) + 1 + \ln 2$

(c) $y = \ln(\sqrt{x^2+3x+2}) + 2 + \ln 2$

(d) $y = \ln(x^2+3x+2) + 1$

(e) $y = \ln(\sqrt{x^2+3x+2}) + 1 + \ln 2$

6. (5 pts.) Evaluate the definite integral

$$\int_1^e \frac{\sqrt{\ln x}}{x}\, dx.$$

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\ln 2$

(d) e^2

(e) 1

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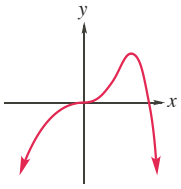
7. (5 pts.) Compute

$$\lim_{x\to\infty}\left(\frac{x+1}{3+4x}\right).$$

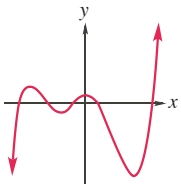
(a) $\frac{1}{4}$ (b) 0 (c) $\frac{1}{3}$ (d) ∞ (e) $\frac{4}{3}$

8. (5 pts.) Which of the following graphs could represent the function $ax^4+bx^3+cx^2+dx+e$ where a, b, c, d, e are constants and $a < 0$? [Hint: pay attention to the condition $a < 0$, and think about what happens as x approaches ∞ or $-\infty$.]

(a)



(b)



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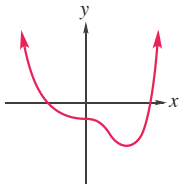
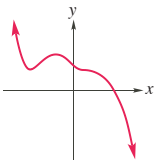
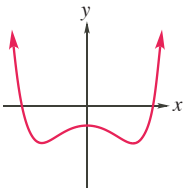


(d)



(e)



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9. (5 pts.) The half-life of carbon-14 is approximately 5730 years. This means that if the initial amount is A_0 then the amount present after t years is

$$A_0 \left(\frac{1}{2}\right)^{\frac{t}{5730}}.$$

How long does it take an initial amount of carbon-14 to decrease to one fifth of the original amount?

(a) $\dfrac{\ln 2}{\ln (5/4)}$ (b) $5730 \cdot \dfrac{\ln 5}{\ln 2}$ (c) $\dfrac{\ln 2}{5730}$

(d) $5730 \cdot \dfrac{\ln (5/4)}{\ln 2}$ (e) $\dfrac{\ln 5}{5730}$

10. (5 pts.) Which equation below describes the tangent line to the graph of $f(x) = \ln(\ln x)$ at the point $(e, 0)$?

(a) $y = x - e$ (b) $y = 0$ (c) $y = e(x - e)$

(d) $y = \frac{1}{e}(x - e)$ (e) $x = e$ (vertical line)

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11. (5 pts.) John invests some money into an account, and learns that the amount in the account after t years is given by the formula

$$A(t) = 1,000e^{0.05t}.$$

What is the *average* amount in the account over the first three years?

(a) $\frac{1}{3} \cdot 20,000[e^{0.15} - 1]$ (b) $\frac{1}{3} \cdot 20,000e^{0.15}$ (c) $\frac{1}{3} \cdot 20,000$

(d) $\frac{1}{3} \cdot 1,000[e^{0.15} - 1]$ (e) $\frac{1}{3} \cdot 1,000e^{0.15}$

12. (5 pts.) Find the total **area** between the x-axis and the graph of $y = x^2 - 1$ from $x = 0$ to $x = 3$. [Hint: this function is not non-negative, so treat areas below the x-axis separately.]

(a) 6 (b) $\frac{44}{3}$ (c) $\frac{20}{3}$ (d) 8 (e) $\frac{22}{3}$

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13. (5 pts.) A ball is thrown into the air and its height in feet after t seconds is given by $h(t) = -16t^2 + 32t + 48$. After how many seconds does the ball reach its peak?

(a) 2 (b) 3 (c) 4 (d) 1 (e) 5

14. (5 pts.) What are the critical points of

$$f(x) = \frac{x}{x^2 + 1}$$

(a) There are none.

(b) $x = -1,\ x = 1$

(c) $x = 0$

(d) $x = -1, x = 0, x = 1$

(e) $x = 0, x = 1$

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15. (5 pts.) Let $f(x)$ be a function whose **derivative** is $f'(x) = x(x-1)^2(x+1)$. Where are the local extrema of $f(x)$?

(a) local minimum at $x = -1$ and local maximum at $x = 0$

(b) local maximum at $x = -1$ and local minimum at $x = 0$

(c) local maximum at $x = -1$ and $x = 1$, and local minimum at $x = 0$

(d) local minimum at $x = -1$ and $x = 1$, and local maximum at $x = 0$

(e) local maximum at $x = -1$ and no local minimum

16. (5 pts.) An automobile company finds that their marginal profit is given by the function $P' = x^2 + 3x$, where x is the number of cars produced per week (in units of one thousand) and $P(x)$ is given in dollars. What is the total change in profit (in dollars) as they raise production from $x = 0$ to $x = 6$ thousand cars per week?

(a) 40 (b) 55 (c) 60 (d) 24 (e) 126

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17. (5 pts.) How many inflection points does $\frac{1}{2}x^4 - \frac{1}{5}x^6$ have?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

18. (5 pts.) Using logarithmic differentiation, find $\dfrac{dy}{dx}$ for

$$y = (2x+1)^4(3x+1)^5(4x+1)^6.$$

[Hint: don't forget the chain rule, and read the different answers carefully!]

(a) $\left(\dfrac{4}{2x+1} + \dfrac{5}{3x+1} + \dfrac{6}{4x+1}\right)(2x+1)^4(3x+1)^5(4x+1)^6$

(b) $\left(\dfrac{8}{2x+1} \cdot \dfrac{15}{3x+1} \cdot \dfrac{24}{4x+1}\right)(2x+1)^4(3x+1)^5(4x+1)^6$

(c) $\left(\dfrac{4}{2x+1} \cdot \dfrac{5}{3x+1} \cdot \dfrac{6}{4x+1}\right)(2x+1)^4(3x+1)^5(4x+1)^6$

(d) $\left(\dfrac{8}{2x+1} + \dfrac{15}{3x+1} + \dfrac{24}{4x+1}\right)(2x+1)^4(3x+1)^5(4x+1)^6$

(e) $\dfrac{4}{2x+1} + \dfrac{5}{3x+1} + \dfrac{6}{4x+1}$

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19. (5 pts.) If $y = \ln\left(\dfrac{11}{x+5}\right)$, solve for x in terms of y.

(a) $x = \dfrac{11}{e^y} - 5$ (b) $x = 6 - e^y$ (c) $x = 16 - e^y$

(d) $x = e^{\frac{y+5}{11}}$ (e) $x = e^{\frac{11-5y}{y}}$

20. (5 pts.) What is the domain of the function $f(x) = \dfrac{\sqrt{x}}{x^5 - 32}$?

(a) $(0, 2) \cup (2, 32) \cup (32, \infty)$ (b) $[0, 2) \cup (2, \infty)$

(c) $[0, 2) \cup (2, 32) \cup (32, \infty)$ (d) $[0, 32) \cup (32, \infty)$

(e) $(-\infty, -32) \cup (-32, 0) \cup (0, 32) \cup (32, \infty)$

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21. (5 pts.) Find the derivative of xe^{x^2+1}.

(a) $e^{x^2+1}(2x^2+1)$ (b) $e^{x^2+1}(x+1)$ (c) $2xe^{x^2+1}$

(d) e^{2x} (e) $e^{x^2+1}+xe^{2x}$

22. (5 pts.) To evaluate the definite integral

$$\int_0^4 \frac{dx}{1+\sqrt{x}}$$

you decide to use the u-substitution $u=\sqrt{x}$. When you make this substitution, what does the definite integral become? [Hint: notice that $u^2=x$.]

(a) $\displaystyle\int_0^4 \frac{2u}{1+u}du$ (b) $\displaystyle\int_0^2 \frac{u}{1+u}du$ (c) $\displaystyle\int_0^4 \frac{u}{1+u}du$

(d) $\displaystyle\int_0^4 \frac{u^2}{1+u}du$ (e) $\displaystyle\int_0^2 \frac{2u}{1+u}du$

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23. (5 pts.) Bob invests a certain sum of money into an account giving interest at an annual rate of 5%, compounded **continuously**. How long will it take his investment to **triple** (all answers are in years)?

(a) $e^{0.15t}$

(b) 60

(c) $\dfrac{\ln 3}{0.05}$

(d) $0.05 \ln 3$

(e) Can't answer without knowing how much was invested

24. (5 pts.) Bob's sister, Bobbie, invests a certain sum of money into an account giving interest at an annual rate of 5%, compounded **annually** (i.e. compounded just once a year). How long will it take her investment to **triple** (all answers are in years)?

(a) $(\ln 3)(\ln(1.05))$

(b) $\ln\left(\dfrac{3}{1.05}\right)$

(c) $\dfrac{\ln 3}{\ln(1.05)}$

(d) $\ln(3.15)$

(e) Can't answer without knowing how much was invested

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25. (5 pts.) What are **all** the value(s) of x where the tangent line to $y = x^3 + x$ has slope 13?

(a) $x = 2$

(b) $x = -2, \quad x = 2$

(c) $x = -2, \quad x = 0, \quad x = 2$

(d) $x = 0$

(e) $x = -2\sqrt{3}, \quad x = 2\sqrt{3}$

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