4/8/22 Exam 3 info:

All A's - 90-100

All B's - 80-89

All C's - 60-79

D - 50-59

F - < 50

Median - 91

Mean - 86.8

Solutions will be posted Monday

On Monday you started §6.2, integration by substitution.

Main formula:

To find $\int f(g(x))\,g'(x)\,dx$ you set $u = g(x) \rightsquigarrow \frac{du}{dx} = g'(x)$ so

$du = g'(x)\,dx$

to convert ↑ to $\int f(u)\,du$. Then look for an antiderivative of $f(x)$.

(Reverse of chain rule)

Then substitute back.

Ex $\int x e^{x^2}\,dx$ let $u = x^2$, $du = 2x\,dx$

$$\int x e^{x^2}\,dx = \int e^{x^2} \cdot x\,dx = \int e^u\,du = e^u + C = e^{x^2} + C$$ Check!

Note: there's no good answer for $\int e^{x^2}dx$.

Ex $\int \frac{\ln(2x)}{x}\,dx$ $u = \ln(2x)$, $\frac{du}{dx} = \frac{1}{2x}(2)$ $du = \frac{1}{2x}(2)\,dx = \frac{1}{x}\,dx$

$$\int \frac{\ln(2x)}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\left(\ln(2x)\right)^2 + C$$ check!

Ex $\int \frac{2e^x}{1+e^x}\,dx$ $\qquad u = 1+e^x \qquad \frac{du}{dx} = e^x$

$du = e^x\,dx$

$\int \frac{2}{u}\,du = 2\ln|u| + C = 2\ln(1+e^x) + C$ $\qquad$ Check!

Ex $\int \frac{\sqrt{\ln x}}{x}\,dx$ $\qquad u = \ln x \qquad du = \frac{1}{x}\,dx$

$= \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(\ln x)^{3/2} + C$

Ex $\int x(x-1)^{10}\,dx$ $\qquad u = x-1 \qquad du = dx$

$x = u+1$

$\int (u+1)u^{10}\,du = \int (u^{11} + u^{10})\,du = \frac{1}{12}u^{12} + \frac{1}{11}u^{11} + C$

$= \frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + C$

Ex $\int \frac{e^{-1/x}}{x^2}\,dx$ $\qquad u = \frac{-1}{x} = -x^{-1}$

$du = x^{-2}\,dx = \frac{1}{x^2}\,dx$

$\int e^u\,du = e^u + C = e^{-1/x} + C$

Ex $\int \frac{x^2}{\sqrt{x^3+1}}\,dx$ $\qquad u = x^3+1 \qquad du = 3x^2\,dx \qquad \frac{1}{3}du = x^2\,dx$

$= \frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}(2)u^{1/2} + C = \frac{2}{3}(x^3+1)^{1/2} + C = \frac{2}{3}\sqrt{x^3+1} + C$

Ex $\displaystyle\int \frac{t}{t+1}\,dt = \int \frac{t+1-1}{t+1}\,dt = \int\left(1-\frac{1}{t+1}\right)dt$ $\qquad u = t+1$, $du = dt$

$$= \int\left(1-\frac{1}{u}\right)du = u - \ln|u| + C = t+1-\ln|t+1| + C$$

Ex $\displaystyle\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx$ $\qquad u = 1+\sqrt{x}$

$$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \qquad du = \frac{1}{2\sqrt{x}}\,dx$$

Problem! Not obvious how to convert x to u uniformly.

$$u = 1+\sqrt{x} \longrightarrow \boxed{\sqrt{x} = u-1} \longrightarrow -\sqrt{x} = 1-u \longrightarrow 1-\sqrt{x} = 2-u$$

$$dx = \left(2\sqrt{x}\right)du = 2(u-1)\,du$$

$$\int \frac{(2-u)}{u}\left(2(u-1)\right)du = 2\int \frac{(2-u)(u-1)}{u}\,du = 2\int \frac{2u-2-u^2+4}{u}\,du$$

$$= 2\int \frac{-u^2+3u-2}{u}\,du = 2\int\left(-u+3-\frac{2}{u}\right)du$$

$$= 2\left[-\frac{1}{2}u^2 + 3u - 2\ln(u)\right] + C$$

$$= 2\left[-\frac{1}{2}\left(1+\sqrt{x}\right)^2 + 3\left(1+\sqrt{x}\right) - 2\ln\left(1+\sqrt{x}\right)\right] + C$$

$$= -\left(1+\sqrt{x}\right)^2 + 6\left(1+\sqrt{x}\right) - 4\ln\left(1+\sqrt{x}\right) + C$$

This is a pain to check.