Statistical Design of Experiments Part II
A CVD System Experiment

Joseph J. Nahas
Outline

1. CVD Overview
2. Taguchi L9 Array
3. Experimental Results
4. Building a Model
5. Analysis of Variance (ANOVA)
Experiment on a CVD Deposition System

• **Goal:** Improve the defect density of the

• **Parameters:**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>°C</td>
<td>T0 - 25</td>
<td>T0</td>
<td>T0 + 25</td>
</tr>
<tr>
<td>Pressure</td>
<td>mtorr</td>
<td>P0 - 200</td>
<td>P0</td>
<td>P0 + 200</td>
</tr>
<tr>
<td>Settling Time</td>
<td>min</td>
<td>t0</td>
<td>t0 + 8</td>
<td>t0 + 16</td>
</tr>
<tr>
<td>Cleaning Method</td>
<td>None</td>
<td>CM2</td>
<td>CM3</td>
<td></td>
</tr>
</tbody>
</table>

• Full Factorial would be $3^4 = 81$ Experiments
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Taguchi L9

- $3^{4-3}$ Fractional Factorial Design
- NIST Dataplot Designs

<table>
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<tr>
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<th>X2</th>
<th>X3</th>
<th>X4</th>
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</tbody>
</table>

TAGUCHI L9 ORTHOGONAL DESIGN

NUMBER OF FACTORS = 4
NUMBER OF LEVELS FOR EACH FACTOR = 3
NUMBER OF OBSERVATIONS = 9

REFERENCE--TAGUCHI, SYS. OF EXP. DES., VOL. 2, PAGE 1153.

NOTE--THIS DESIGN IS EQUIVALENT TO A
3**(4-2) FRACTIONAL FACTORIAL DESIGN.

REFERENCE--BARKER, QUALITY BY EXP. DESIGN, PAGES 74-87.

- Each column is orthogonal to the other columns
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Experimental Results

- Because of the large range in defect density, we can use a log.
- Measure = - 10 log (Defect Density^2)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Temperature</th>
<th>Pressure</th>
<th>Settling Time</th>
<th>Cleaning Method</th>
<th>Defect Density</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>n1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>n2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>33</td>
<td>n3</td>
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<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>17</td>
<td>n4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>173</td>
<td>n5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1,700</td>
<td>n6</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>170</td>
<td>n7</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1,700</td>
<td>n8</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3,100</td>
<td>n9</td>
</tr>
</tbody>
</table>
Response to Parameters

• Overall Mean
  – $m = \frac{1}{9} [n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + n9]$
  – $m = \frac{1}{9} [-20 -10 -30 -25 -45 -65 -65 -70] = -41.7$

• Low Temperature
  – $mA1 = \frac{1}{3} [n1 + n2 + n3] = \frac{1}{3} [-20 -10 -30] = -20$

• Medium Temperature
  – $mA2 = \frac{1}{3} [n4 + n5 + n6] = \frac{1}{3} [-25 -45 -65] = -45$

• High Temperature
  – $mA3 = \frac{1}{3} [n7 + n8 + n9] = \frac{1}{3} [-45 -65 -70] = -60$

• Low Pressure
  – $mB1 = \frac{1}{3} [n1 + n4 + n7] = \frac{1}{3} [-20 -25 -45] = -30$

• Medium Pressure
  – $mB2 = \frac{1}{3} [n2 + n5 + n8] = \frac{1}{3} [-10 -45 -65] = -40$

• High Pressure
  – $mB3 = \frac{1}{3} [n3 + n6 + n9] = \frac{1}{3} [-30 -65 -70] = -55$
Parameter Response

- Starting level are underlined.
- Best level are in red.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Temperature</td>
<td>-20</td>
<td>-45</td>
<td>-60</td>
</tr>
<tr>
<td>B Pressure</td>
<td>-30</td>
<td>-40</td>
<td>-55</td>
</tr>
<tr>
<td>C Settling Time</td>
<td>-50</td>
<td>-35</td>
<td>-40</td>
</tr>
<tr>
<td>D Cleaning Method</td>
<td>-45</td>
<td>-40</td>
<td>-40</td>
</tr>
</tbody>
</table>
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Building a Model for the Results

- A simple superposition (additive) model:
  \[ n(A_i, B_j, C_k, D_l) = m + a_i + b_j + c_k + d_l + e_{ijkl} \]
  - \( A \) is temperature, \( B \) is pressure, etc.
  - \( m \) is the overall mean response.
  - \( i, j, k, l \) are 1, 2, or 3, i.e. the levels in the experiment.
  - \( a_1 \) is the differential response to Temperature at the first level
  - \( a_2 \) is the differential response to Temperature at the second level.
  - etc.
  - \( e \) is the error.

- By definition \( a_1, a_2, \) and \( a_3 \) are the deviations from \( m \) caused by the three levels of \( A \). Thus:
  \[ a_1 + a_2 + a_3 = 0 \]

- Similarly:
  \[ b_1 + b_2 + b_3 = 0 \]
  \[ c_1 + c_2 + c_3 = 0 \]
  \[ d_1 + d_2 + d_3 = 0 \]
Look at mA3

\[ mA3 = \frac{1}{3} [n7 + n8 + n9] \]

or

\[ mA3 = \frac{1}{3}[(m+a_3+b_1+c_3+d_2+e_7)+(m+a_3+b_2+c_1+d_3+e_8)\]

\[ + (m+a_3+b_3+c_2+d_1+e_9)] \]

\[ mA3 = m + a_3 + \frac{1}{3}(e_7 + e_8 + e_9) \]

or

\[ a_3 = mA3 - m - \frac{1}{3}(e_7 + e_8 + e_9) \]

if e has a variance of \( \sigma_e^2 \) then \( a_3 \) has an error variance of \( 1/3 \sigma_e^2 \).
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Analysis of Variance

• Decomposition of Parameters affecting the result.
  – Somewhat comparable to Fourier Analysis of a waveform.
    ▪ The nine observations are analogous to the signal.
    ▪ The sum of squared values on n is analogous to the signal power.
    ▪ The overall mean is analogous to the DC part of the signal.
    ▪ The four factors are like four harmonics.
    ▪ The column of the matrix are orthogonal like the harmonics.
Sum of the Squares

- **Grand Sum of Squares**
  - Equivalent to the Total Power
    \[ GSS = n_1^2 + n_2^2 + n_3^2 ... + n_9^2 \]
    \[ = (-20)^2 + (-10)^2 + ... + (-70)^2 \]
    \[ = 19,425 \text{ (dB)}^2 \]

- **Sum of Squares due to mean**
  - Equivalent to DC Power
    \[ SSM = 9 \times m^2 = 9 \times (-41.7)^2 \]
    \[ = 15,625 \text{ (dB)}^2 \]

- **Total Sum of Squares**
  - Equivalent to the AC Power
    \[ SS = \sum_{i=1}^{9}(n_i - m)^2 \]
    \[ = 3,800 \text{ (dB)}^2 \]
Sum of Squares due to A (Temperature)

• Sum of Squares of Deviation from the mean for Temperature:
  \[ SSA = 3 \times (mA1 - m)^2 + 3 \times (mA2 - m)^2 + 3 \times (mA3 - m)^2 \]
  \[ = 3(-20 + 41.7)^2 + 3(-45 + 41.4)^2 + 3(-60 + 41.7)^2 \]
  \[ = 2450 \text{ (dB)}^2 \]

• This explains 65% of the variation:
  \[ \frac{2450}{3800} = 65\% \]
The Settling Time and Cleaning Method are literally in the noise.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Temperature</td>
<td>2</td>
<td>2450</td>
<td>1225</td>
<td>64%</td>
</tr>
<tr>
<td>B Pressure</td>
<td>2</td>
<td>950</td>
<td>475</td>
<td>25%</td>
</tr>
<tr>
<td>C Settling Time</td>
<td>2</td>
<td>350</td>
<td>175</td>
<td>9%</td>
</tr>
<tr>
<td>D Cleaning Method</td>
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<td>50</td>
<td>25</td>
<td>1%</td>
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<tr>
<td>Error</td>
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</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>3800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
With the Settling Time and Cleaning Method as part of the noise, the Variance of the Noise is $\sigma_e^2 = 100 \text{ dB}^2$.

The variance for each measurement is $1/3 \sigma_e^2$.

So $\sigma_m = (100/3)^{1/2} = 5.75 \text{ dB}$

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<td>B Pressure</td>
<td>2</td>
<td>950</td>
<td>475</td>
<td>25%</td>
<td>4.75</td>
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<tr>
<td>C Settling Time</td>
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<td>0</td>
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</tr>
<tr>
<td>D Cleaning Method</td>
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<td>0%</td>
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<tr>
<td>Error</td>
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<td>100</td>
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<tr>
<td>Total</td>
<td>8</td>
<td>3800</td>
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**Parameter Response**

- Starting level are **underlined**.
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<td>-40</td>
</tr>
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</table>
Response with Error Bars