Statistical Design of Experiments Part II
A CVD System Experement

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Outline

1. CVD Overview
2. Taguchi L9 Array
3. Experimental Results
4. Building a Model
5. Analysis of Variance (ANOVA)

DoE Example

Experiment on a CVD Deposition System

- **Goal: Improve the defect density of the**
- **Parameters:**

		Levels		
Parameters	**Unit**	**1**	**2**	**3**
Temperature	C	T0 - 25	T0	T0 + 25
Pressure	mtorr	P0 - 200	P0	P0 + 200
Settling Time	min	t0	t0 + 8	t0 + 16
Cleaning Method		None	CM2	CM3

- **Full Factorial would be $3^4 = 81$ Experiments**

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Taguchi L9

- 3^{4-3} Fractional Factoral Design
- NIST Dataplot Designs
  - http://www.itl.nist.gov/div898/software/dataplot/designs.htm

X1	X2	X3	X4
1	1	1	1
1	2	2	2
1	3	3	3
2	1	2	3
2	2	3	1
2	3	1	2
3	1	3	2
3	2	1	3
3	3	2	1

```
TAGUCHI L9 ORTHOGONAL DESIGN
NUMBER OF FACTORS                = 4
NUMBER OF LEVELS FOR EACH FACTOR = 3
NUMBER OF OBSERVATIONS           = 9
REFERENCE--TAGUCHI, SYS. OF EXP. DES., VOL. 2, PAGE 1153.
NOTE--THIS DESIGN IS EQUIVALENT TO A
      3**(4-2) FRACTIONAL FACTORIAL DESIGN.
      REFERENCE--BARKER, QUALITY BY EXP. DESIGN, PAGES 74-87.
```

- Each column is orthogonal to the other columns

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Experimental Results

- Because of the large range in defect density, we can use a log.
- Measure = - 10 log (Defect Density^2)

	1	2	3	4			
Experiment	Temperature	Pressure	Settling Time	Cleaning Method	Defect Density	Measure	
	A	B	C	D	d/cm2	symbol	dB
1	1	1	1	1	10	n1	-20
2	1	2	2	2	3	n2	-10
3	1	3	3	3	33	n3	-30
4	2	1	2	3	17	n4	-25
5	2	2	3	1	173	n5	-45
6	2	3	1	2	1,700	n6	-65
7	3	1	3	2	170	n7	-45
8	3	2	1	3	1,700	n8	-65
9	3	3	2	1	3,100	n9	-70

Response to Parameters

- Overall Mean
  - m = 1/9 [n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + n9]
  - m = 1/9 [-20 -10 -30 -25 -45 -65 -45 -65 -70] = -41.7
- Low Temperature
  - mA1 = 1/3 [n1 + n2 + n3] = 1/3 [-20 -10 -30] = -20
- Medium Temperature
  - mA2 = 1/3 [n4 + n5 + n6] = 1/3 [-25 -45 -65] = -45
- High Temperature
  - mA3 = 1/3 [n7 + n8 + n9] = 1/3 [-45 -65 -70] = -60
- Low Pressure
  - mB1 = 1/3 [n1 + n4 + n7] = 1/3 [-20 -25 -45] = -30
- Medium Pressure
  - mB2 = 1/3 [n2 + n5 + n8] = 1/3 [-10 -45 -65] = -40
- High Pressure
  - mB3 = 1/3 [n3 + n6 + n9] = 1/3 [-30 –65 –70] = -55

Parameter Response

- Starting level are <u>underlined.</u>
- Best level are in red.

	Level		
	1	2	3
A Temperature	-20	<u>-45</u>	-60
B Pressure	-30	<u>-40</u>	-55
C Settling Time	<u>-50</u>	-35	-40
D Cleaning Method	<u>-45</u>	-40	-40

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Building a Model for the Results

- A simple superposition (additive) model:

$$n(A_i, B_j, C_k, D_l) = m + a_i + b_j + c_k + d_l + e_{ijkl}$$

  – A is temperature, B is pressure, etc.
  – m is the overall mean response.
  – i, j, k, l are 1, 2, or 3, i.e. the levels in the experiment.
  – a_1 is the differential response to Temperature at the first level
  – a_2 is the differential response to Temperature at the second level.
  – etc.
  – e is the error.
- By definition a_1, a_2, and a_3 are the deviations from m caused by the three levels of A. Thus:

$$a_1 + a_2 + a_3 = 0$$

- Similarly:

$$b_1 + b_2 + b_3 = 0$$
$$c_1 + c_2 + c_3 = 0$$
$$d_1 + d_2 + d_3 = 0$$

Look at mA3

$$mA3 = 1/3\ [n7 + n8 + n9]$$

or

$$mA3 = 1/3[(m+a_3+b_1+c_3+d_2+e_7)+(m+a_3+b_2+c_1+d_3+e_8)$$
$$+(m+a_3+b_3+c_2+d_1+e_9)]$$

$$mA3 = m + a_3 + 1/3(e_7 + e_8 + e_9)$$

or

$$a_3 = mA3 – m – 1/3(e_7 + e_8 + e_9)$$

if e has a variance of σ_e^2 then a_3 has an error variance of 1/3 σ_e^2.

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Analysis of Variance

- **Decomposition of Parameters affecting the result.**
  - **Somewhat comparable to Fourier Analysis of a waveform.**
    - **The nine observations are analogous to the signal.**
    - **The sum of squared values on n is analogous to the signal power.**
    - **The overall mean is analogous to the DC part of the signal.**
    - **The four factors are like four harmonics.**
    - **The column of the matrix are orthogonal like the harmonics.**

Sum of the Squares

- **Grand Sum of Squares**
  - Equivalent to the Total Power

$$GSS = n_1^2 + n_2^2 + n_3^2 \ldots + n_9^2$$
$$= (-20)^2 + (-10)^2 + \ldots + (-70)^2$$
$$= 19{,}425\ (dB)^2$$

- **Sum of Squares due to mean**
  - Equivalent to DC Power

$$SSM = 9 * m^2 = 9 * (-41.7)^2$$
$$= 15{,}625\ (dB)^2$$

- **Total Sum of Squares**
  - Equivalent to the AC Power

$$SS = sum_{(i = 1 \text{ to } 9)}(n_i - m)^2$$
$$= 3{,}800\ (dB)^2$$

Sum of Squares due to A (Temperature)

- **Sum of Squares of Deviation from the mean for Temperature:**

$$SSA = 3 * (mA1 - m)^2 + 3 * (mA2 - m)^2 + 3 * (mA3 - m)^2$$

$$= 3(-20 + 41.7)^2 + 3(-45 + 41.4)^2 + 3(-60 + 41.7)^2$$

$$= 2450\ (dB)^2$$

- **This explains 65% of the variation:**

2450/3800 = 65%

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ANOVA Table 1

- The Settling Time and Cleaning Method are literally in the noise.

	Degrees of Freedom	Sum of Squares	Mean Square	Percent
A Temperature	2	2450	1225	64%
B Pressure	2	950	475	25%
C Settling Time	2	350	175	9%
D Cleaning Method	2	50	25	1%
Error	0	0		
Total	8	3800		

ANOVA Table 2

- With the Settling Time and Cleaning Method as part of the noise, the Variance of the Noise is σ_e^2 = 100 dB^2.
- The variance for each measurement is 1/3 σ_e^2.
- So $\sigma_m = (100/3)^{1/2}$ = 5.75 dB
- 2 σ_m = 11.5 dB

	Degrees of Freedom	Sum of Squares	Mean Square	Percent	F
A Temperature	2	2450	1225	64%	12.25
B Pressure	2	950	475	25%	4.75
C Settling Time	0	0	0	0%	
D Cleaning Method	0	0	0	0%	
Error	4	400	100		
Total	8	3800			

Parameter Response

- Starting level are underlined.
- Best level are in red.

	Level		
	1	2	3
A Temperature	-20	-45	-60
B Pressure	-30	-40	-55
C Settling Time	-50	-35	-40
D Cleaning Method	-45	-40	-40

Response with Error Bars