

# A Case For Amplify-Forward Relaying in the Block-Fading Multi-Access Channel

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## Abstract

This paper demonstrates the significant gains that multi-access users can achieve from *sharing* a single amplify-forward relay in slow fading environments. The proposed protocol, namely multi-access amplify-forward, allows for a low-complexity relay and achieves the optimal diversity-multiplexing trade-off at high multiplexing gains. Analysis of the protocol further reveals that it outperforms both the compress-forward strategy at low multiplexing gains and the dynamic decode-forward protocol at high multiplexing gains. An interesting feature of the proposed protocol is that, at high multiplexing gains, it resembles a multiple-input single-output system, and at low multiplexing gains, it provides each user with the same diversity-multiplexing trade-off as if there is no contention for the relay from the other users.

## I. INTRODUCTION

### A. Motivation

In recent years, cooperative communications has received significant interest (e.g., [1]–[7]) as a means of providing spatial diversity for applications in which temporal, spectral, and antenna diversity are limited by delay, bandwidth, and terminal size constraints, respectively. Cooperative techniques offer diversity by enabling users to utilize one another’s resources such as antennas, power, and bandwidth. As a consequence, most cooperative protocols share the characteristic

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that they require substantial coordination among the users. In a wireless setting, establishing this level of user cooperation may be impractical due to cost and complexity considerations. Inspired by this observation, the current paper focuses on an alternative architecture, namely, the multi-access relay channel (MARC) [4], [8] and proposes a strategy called the multi-access amplify-forward (MAF) that allows the users to operate as if in a normal (non-cooperative) multi-access channel. In this system, the users need not be aware of the existence of the relay, *i.e.*, all cost and complexity of exploiting cooperative diversity is placed in the relay and destination. Such an architecture may be suitable for infrastructure networks, in which the relay and destination correspond respectively to a relay station and a base station deployed and managed by the service provider. It is worth noting that since a single relay is *shared* by multiple users in the MARC, the extra cost of adding the relay is amortized across many users and may thus be more acceptable, especially as the number of users in the system grows. Thus, this approach facilitates a graceful transition, both technologic as well as economic, from existing systems to cooperative ones.

### *B. Related Research*

This section provides a brief review of the related research. The MARC was first introduced in [8] as a model for topologies in which multiple sources communicate with a single destination in the presence of a relay. Information-theoretic treatment of the MARC has focused on two aspects, namely, the capacity region and the diversity-multiplexing trade-off (DMT) [9]. Using a partial decode-forward strategy, [7] compares the additive white Gaussian noise (AWGN) MARC with cooperative multi-access communications and shows that the former achieves higher rates than the latter. Using a full-duplex relay, [4] shows that a decode-forward strategy achieves the capacity of the AWGN MARC assuming the relay is geometrically close to the sources. For the general MARC, however, the optimum relaying strategy in terms of achieving the capacity region remains unknown.

The DMT of the MARC<sup>1</sup> is studied in [5] and [6]. In [5], the dynamic decode-forward (DDF) strategy is applied to the MARC. In DDF, the relay does not decode until it collects sufficient information for error-free detection of the message. The relay then re-encodes the message and transmits it in the remainder of the coding interval. For the MARC, DDF is shown to achieve

<sup>1</sup>The rest of the paper focuses on the block fading scenario; the term “MARC” therefore refers to the “block-fading MARC”.

the optimal DMT for low multiplexing gains. However, for high multiplexing gains, it becomes suboptimal. Another relaying strategy for the MARC is compress-forward (CF) [10], [6]. In CF, the relay employs source coding with side information, *i.e.*, Wyner-Ziv coding, to compress its received signal and forward it to the destination. CF achieves the optimal DMT for high multiplexing gains [6], but analysis so far suggests that it suffers from significant diversity loss for low multiplexing gains.

### C. Summary of Results

This section summarizes our contributions. Assuming a half-duplex relay, we propose a protocol, *i.e.*, MAF, for the MARC and demonstrate the significant gains that it provides to multi-access users. Since MAF is essentially an amplify-forward (AF) protocol, the relay does not require complicated decoding and encoding. By contrast, some of the previously proposed MARC protocols, such as DDF [5] or CF [6], require complex signal processing at the relay. The benefits of the proposed protocol are not limited to complexity aspects. As argued in the sequel, the MAF protocol outperforms both the DDF protocol in the high multiplexing regime and the CF protocol in the low multiplexing regime. In particular, MAF achieves the optimal DMT of the MARC for multiplexing gains greater than  $2/3$ . This is somewhat counterintuitive considering the fact that AF relay protocols generally suffer from a significant performance loss in the high multiplexing regime [2], [3]. It is also worth noting that, for low multiplexing gains, each user in the MAF protocol receives the same benefit from the relay as if it is the only user present, *i.e.*, the advantage of using a single relay does *not* vanish as the number of users grows. Overall, MAF provides a nice balance between complexity and performance.

## II. BACKGROUND

### A. Notation

In this paper, random variables and random vectors are denoted using the sans serif (*e.g.*,  $x$ ) and bold sans serif (*e.g.*,  $\mathbf{x}$ ) fonts, respectively. Calligraphic letters denote events or sets (*e.g.*,  $\mathcal{S}$ ), and  $x^+$  means  $\max\{x, 0\}$ . Exponential equality is denoted by  $\doteq$ , *i.e.*,  $f(\rho) \doteq \rho^v$  when

$$\lim_{\rho \rightarrow \infty} \frac{\log f(\rho)}{\log \rho} = v.$$

In this expression,  $v$  is called the *exponential order* of  $f(\rho)$ . The relations  $\stackrel{\leq}{\doteq}$  and  $\stackrel{\geq}{\doteq}$  are defined similarly.

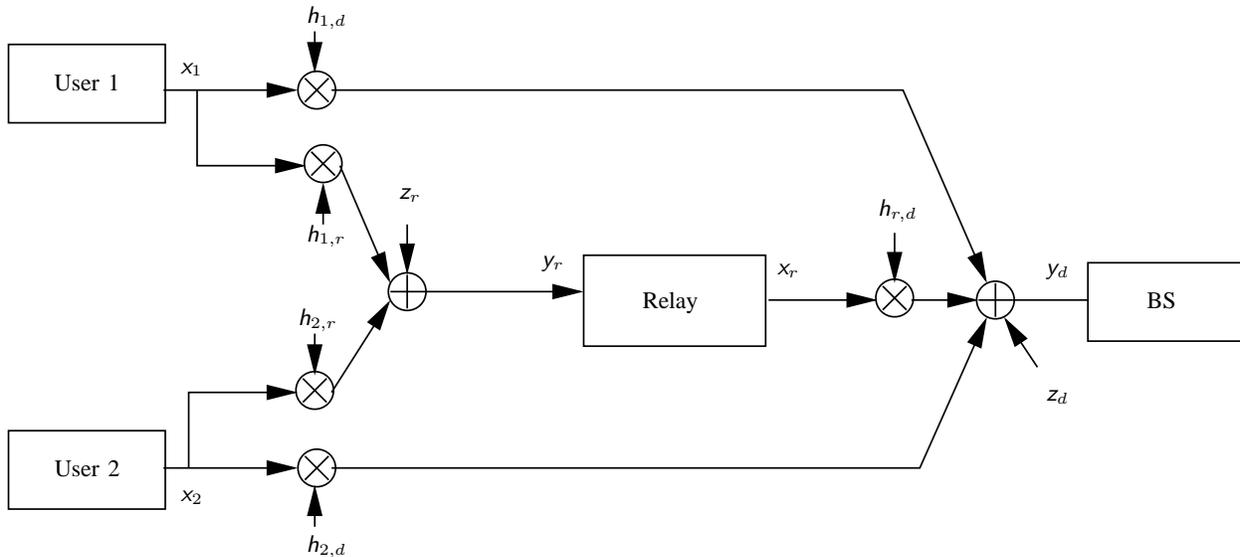


Fig. 1. Multi-access relay channel (MARC) with two users, multiplicative fading, and additive noise.

## B. Channel

The MARC is distinguished from the standard multi-access channel by the existence of one or more relays solely intended to facilitate communication between the users and the destination. For simplicity of presentation, this paper focuses on the case of two users and one relay as shown in Fig. 1.

All wireless links are assumed to be frequency non-selective and Rayleigh block fading, *i.e.*, the fading coefficients remain constant within a block of  $l$  symbols, but change independently from one block to the other according to a circularly symmetric complex Gaussian distribution with zero mean and unit variance. The terminals are assumed to be far enough from one another such that the fading coefficients of different links are independent. Moreover, the block length  $l$  is assumed to be long enough such that the channel state information (CSI) can be tracked at the receiving end, though it is not available to or otherwise not exploited by the transmitting end. It is assumed that the destination has knowledge of all CSI, including those of the user-relay links. The variance of the AWGN is taken to be unity.

### C. Analysis Method

In order to characterize the performance of the proposed protocol in the high SNR regime, the DMT is adopted as the performance metric [9]. This paper focuses on the symmetric case, *i.e.*, the two users transmit their messages at the same data rate of  $R/2$  bits per channel use (bpcu). Furthermore, the two users and the relay use the same transmission power  $\rho$  since power allocation in the absence of transmitter CSI does not improve a protocol's DMT performance. Let  $\mathcal{C}(\rho) = \{\mathcal{C}_1(\rho), \mathcal{C}_2(\rho)\}$  denote a family of codes indexed by SNR  $\rho$ , such that user  $i$ 's codebook  $\mathcal{C}_i(\rho)$  has data rate  $R(\rho)/2$  and codelength  $l$ . Also, let  $P_{\mathcal{E}}(\rho, R(\rho))$  denote the error probability of the joint ML decoder at the base station. For this setup, the multiplexing gain  $r$  and the diversity gain  $d$  are defined as

$$r := \lim_{\rho \rightarrow \infty} \frac{R(\rho)}{\log \rho}, \quad d := \lim_{\rho \rightarrow \infty} -\frac{\log P_{\mathcal{E}}(\rho, R(\rho))}{\log \rho}.$$

A protocol's DMT views the diversity gain  $d$  as a function of multiplexing gain  $r$ , *i.e.*,  $d(r)$  vs.  $r$ .

### III. MULTI-ACCESS AMPLIFY-FORWARD

This section describes the proposed MAF protocol. In MAF, the relay listens to the two users during the first half of the block; then, in the second half of the block, it simply amplifies and broadcasts the signal it received in the first half. The two users both continue transmitting their messages throughout the entire block. During the first half of the block, the equivalent channels seen by the destination and the relay are

$$y_d[j] = \sum_{i=1}^2 h_{i,d} x_i[j] + z_d[j], \quad (1)$$

$$y_r[j] = \sum_{i=1}^2 h_{i,r} x_i[j] + z_r[j], \quad (2)$$

respectively, where  $1 \leq j \leq l/2$  denotes the time index;  $h_{i,d}$  and  $h_{i,r}$  denote the fading coefficients of the user  $i$ -destination and user  $i$ -relay links, respectively; and  $x_i$  denotes the signal transmitted by user  $i$ . Likewise, the equivalent channel seen by the destination during the second half of the block is

$$y_d[j] = \sum_{i=1}^2 h_{i,d} x_i[j] + h_{r,d} x_r[j] + z_d[j] \quad \text{for } l/2 < j \leq l, \quad (3)$$

where  $h_{r,d}$  denotes the fading coefficient of the relay-destination link, and  $x_r$  denotes the signal transmitted by the relay. Note that

$$x_r[j] = by_r[j - l/2] \quad \text{for } l/2 < j \leq l,$$

where  $b$  denotes the relay's amplification coefficient. In practice,  $b$  should be chosen to, *e.g.*, minimize the outage probability at the target data rate and SNR, subject to the relay's transmission power constraint, *i.e.*,

$$|b|^2 \leq \frac{\rho}{\sum_{i=1}^2 |h_{i,r}|^2 \rho + 1}. \quad (4)$$

However, as shown in [3], the particular value of  $b$  does not affect the DMT of the protocol as long as its exponential order remains zero (which is guaranteed by (4)). Therefore, the optimization of  $b$  is not further pursued in this paper. Note that the base station needs to know the value of  $b$  to decode the two messages. As the last comment, in the single user scenario, the MAF protocol reduces to the non-orthogonal amplify-forward (NAF) protocol of [3], [11].

As mentioned earlier, one of the features of MAF is that it allows the users to operate as if in a normal MAC. In fact, the users may not be aware of the existence of the relay. Therefore, they simply use the capacity-achieving codebook for a standard MAC, *i.e.*, each codebook consists of i.i.d complex Gaussian random variables. Such inputs may not be optimal in terms of capacity or outage probability for MAF, due to the correlation that exists between the relay's signal and those of the users. However, as the following theorem shows, Gaussian inputs are optimal in terms of DMT at high multiplexing gains.

*Theorem 1:* For the symmetric MARC with two users and one relay, the DMT of the MAF protocol is given by

$$d_{MAF}(r) = \begin{cases} 2 - \frac{3r}{2}, & \text{for } 0 \leq r \leq \frac{2}{3} \\ 3(1 - r), & \text{for } \frac{2}{3} \leq r \leq 1 \end{cases}. \quad (5)$$

*Proof:* The proof uses the machinery of Theorem 2 in [12] and Lemma 2 in [5] as well as some of the techniques of Theorem 3 in [3]. Therefore, we only provide a sketch of the main steps involved and focus on the novel parts.

Following the outline of [12] and [5], we split the joint error event  $\mathcal{E}$  into mutually exclusive error events  $\mathcal{E}_1$ ,  $\mathcal{E}_2$  and  $\mathcal{E}_3$ , *i.e.*,

$$\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}_3, \quad (6)$$

where  $\mathcal{E}_1$  ( $\mathcal{E}_2$ ) represents the error event that only user 1 (2) is detected in error, and  $\mathcal{E}_3$  represents the error event that both of the users are detected in error. Applying the union bound to (6) yields

$$P_{\mathcal{E}} \leq P_{\mathcal{E}_1} + P_{\mathcal{E}_2} + P_{\mathcal{E}_3}. \quad (7)$$

$P_{\mathcal{E}_1}$ ,  $P_{\mathcal{E}_2}$  and  $P_{\mathcal{E}_3}$  are bounded by first eliminating the contribution of the correctly decoded user, if any, from the received signal, and then bounding the corresponding pairwise error probabilities for the remaining user(s) [3], *i.e.*,

$$P_{\mathcal{E}_i|\mathbf{h}} \leq \left[ 1 + \frac{\rho}{2} |h_{i,d}|^2 + \frac{\rho}{2} \frac{|h_{i,d}|^2 + |bh_{r,d}h_{i,r}|^2 + \frac{\rho}{2} |h_{i,d}|^4}{1 + |bh_{r,d}|^2} \right]^{-l/2} \rho^{rl/2} \quad \text{for } i = 1, 2 \quad \text{and} \quad (8)$$

$$P_{\mathcal{E}_3|\mathbf{h}} \leq \left[ 1 + \frac{\rho}{2} (|h_{1,d}|^2 + |h_{2,d}|^2) + \frac{\rho (|h_{1,d}|^2 + |h_{2,d}|^2) + |bh_{r,d}|^2 (|h_{1,r}|^2 + |h_{2,r}|^2)}{1 + |bh_{r,d}|^2} \right. \\ \left. + \frac{\rho^2 (|h_{1,d}|^2 + |h_{2,d}|^2)^2 + |bh_{r,d}|^2 |h_{1,d}h_{2,r} - h_{2,d}h_{1,r}|^2}{1 + |bh_{r,d}|^2} \right]^{-l/2} \rho^{rl}, \quad (9)$$

where  $\mathbf{h} = [h_{1,r}, h_{2,r}, h_{r,d}, h_{1,d}, h_{2,d}]$ . Averaging (8) over the ensemble of channel realizations results in

$$P_{\mathcal{E}_i} \leq \rho^{-[(1-\frac{r}{2})^+ + (1-r)^+]} \quad \text{for } i = 1, 2. \quad (10)$$

However, averaging (9) to bound  $P_{\mathcal{E}_3}$  is not straightforward due to the term

$$|h_{1,d}h_{2,r} - h_{2,d}h_{1,r}|^2,$$

which involves subtraction. To circumvent this problem, define

$$\Theta := \frac{h_{2,r}h_{1,d} - h_{1,r}h_{2,d}}{\sqrt{|h_{1,r}|^2 + |h_{2,r}|^2}} \quad \text{and} \quad (11)$$

$$\Omega := \frac{h_{1,r}^*h_{1,d} + h_{2,r}^*h_{2,d}}{\sqrt{|h_{1,r}|^2 + |h_{2,r}|^2}}. \quad (12)$$

It is then straightforward to see that *conditioned* on  $h_{1,r}$  and  $h_{2,r}$ ,  $\Theta$  and  $\Omega$  are two complex Gaussian random variables with zero mean and unit variance. Furthermore,  $E\{\Theta\Omega^* | h_{1,r}, h_{2,r}\} = 0$ , meaning that  $\Theta$  and  $\Omega$  are conditionally uncorrelated and therefore independent. Realizing that,

$$|\Theta|^2 + |\Omega|^2 = |h_{1,d}|^2 + |h_{2,d}|^2,$$

(9) can be written as

$$P_{\mathcal{E}_3|\mathbf{h}} \leq \left[ 1 + \frac{\rho}{2}(|\Theta|^2 + |\Omega|^2) + \frac{\rho}{2} \frac{(|\Theta|^2 + |\Omega|^2) + |bh_{r,d}|^2 (|h_{1,r}|^2 + |h_{2,r}|^2)}{1 + |bh_{r,d}|^2} + \frac{\rho^2}{4} \frac{(|\Theta|^2 + |\Omega|^2)^2 + |bh_{r,d}|^2 (|h_{1,r}|^2 + |h_{2,r}|^2) |\Theta|^2}{1 + |bh_{r,d}|^2} \right]^{-l/2} \rho^{rl}. \quad (13)$$

Note that since  $\Theta$ ,  $\Omega$ ,  $h_{1,r}$  and  $h_{2,r}$  are correlated, the techniques of [3] cannot directly be applied to average (13). However, by averaging in two steps, *i.e.*, fixing  $h_{1,r}$  and  $h_{2,r}$  and taking the conditional average with respect to  $\Theta$ ,  $\Omega$  and  $h_{r,d}$ , and then taking the average with respect to  $h_{1,r}$  and  $h_{2,r}$ ,  $P_{\mathcal{E}_3}$  can be bounded. More specifically, conditioned on  $h_{1,r}$  and  $h_{2,r}$ , (13) can be averaged with respect to  $\Theta$ ,  $\Omega$  and  $h_{r,d}$  to obtain,

$$P_{\mathcal{E}_3|h_{1,r},h_{2,r}} \leq \rho^{-d_{\mathcal{E}_3|h_{1,r},h_{2,r}}}, \quad (14)$$

where

$$d_{\mathcal{E}_3|h_{1,r},h_{2,r}} = \begin{cases} 2(1-r)^+ & \text{for } \min\{v_{1,r}, v_{2,r}\} > (1-r)^+ \\ [3(1-r) - \min\{v_{1,r}, v_{2,r}\}]^+ & \text{for } 0 \leq \min\{v_{1,r}, v_{2,r}\} \leq (1-r)^+ \end{cases} \quad (15)$$

and  $v_{i,r}$  is the exponential order of  $|h_{i,r}|^2$  for  $i = 1, 2$ . Next, averaging (14) with respect to  $v_{1,r}$  and  $v_{2,r}$  gives

$$P_{\mathcal{E}_3} \leq \rho^{-3(1-r)^+}. \quad (16)$$

Finally, (16) together with (10) and (7) results in (5), and thus completes the proof.  $\blacksquare$

#### IV. DISCUSSION

For purposes of comparison, an upper bound on the achievable DMT for the symmetric MARC is first provided, along with the DMT's of the DDF and CF protocols. For the symmetric MARC with two users and one relay, an upper bound on the achievable DMT is [5]

$$d_{MARC}(r) \leq \begin{cases} 2-r, & \text{for } 0 \leq r \leq \frac{1}{2} \\ 3(1-r), & \text{for } \frac{1}{2} \leq r \leq 1 \end{cases}. \quad (17)$$

On the other hand, the DMT of DDF is [5]

$$d_{DDF}(r) = \begin{cases} 2-r, & \text{for } 0 \leq r \leq \frac{1}{2} \\ 3(1-r), & \text{for } \frac{1}{2} \leq r \leq \frac{2}{3}, \\ \frac{2(1-r)}{r}, & \text{for } \frac{2}{3} \leq r \leq 1 \end{cases}, \quad (18)$$

and that of CF is [6]

$$d_{CF}(r) = \begin{cases} 2(1-r), & \text{for } 0 \leq r \leq \frac{2}{3} \\ 1 - \frac{r}{2}, & \text{for } \frac{2}{3} \leq r \leq \frac{4}{5} \\ 3(1-r), & \text{for } \frac{4}{5} \leq r \leq 1 \end{cases}. \quad (19)$$

It is noted that the DMT of (19) is achieved by two different CF operations [6], *i.e.*, mobile stations transmit in a time sharing fashion for  $0 \leq r \leq \frac{2}{3}$ , but transmit simultaneously for  $\frac{2}{3} \leq r \leq 1$ . To highlight the advantage gained from adding a single relay, the DMT of a symmetric MAC with two users is also provided [12], *i.e.*,

$$d_{MAC}(r) = \begin{cases} 1 - \frac{r}{2}, & \text{for } 0 \leq r \leq \frac{2}{3} \\ 2(1-r), & \text{for } \frac{2}{3} \leq r \leq 1 \end{cases}. \quad (20)$$

Fig. 2 shows these trade-offs, along with the trade-off for MAF as given by (5). Inspecting these results and Fig. 2, the following observations can be made:

- 1) The MAF protocol achieves the optimal DMT for  $2/3 \leq r \leq 1$ . In fact, over this range of multiplexing gains, MAF behaves like a MISO system with three transmit antennas and one receive antenna.
- 2) MAF outperforms the CF protocol in terms of DMT, *i.e.*,  $d_{MAF}(r) \geq d_{CF}(r), \forall r$ . Relative to MAF, CF suffers from a loss in diversity gain at low multiplexing gains. Compared to CF, MAF enjoys another advantage of lower complexity at the relay.
- 3) It is interesting to observe that for  $2/3 \leq r \leq 1$ , MAF outperforms DDF in terms of the DMT, despite the fact that AF protocols generally suffer from a significant performance loss relative to DDF in the single user relay channel [2], [3]. The advantage of MAF over DDF at high multiplexing gains can be attributed to the fact that in this operating regime, DDF requires the relay to spend a large percentage of time decoding the two users' messages. As a result, the relay may not have enough time to re-transmit the decoded information. On the other hand, it has been observed that the performance of AF protocols improves in other multi-user scenarios, *e.g.*, for the cooperative multi-access channel, the nonorthogonal amplify-forward (CMA-NAF) protocol achieves the optimal DMT [3].
- 4) In the regime of  $2/3 \leq r \leq 4/5$ , neither DDF nor CF is optimal, but MAF is optimal. To the best of our knowledge, MAF is the only protocol that achieves the optimal DMT in this regime.

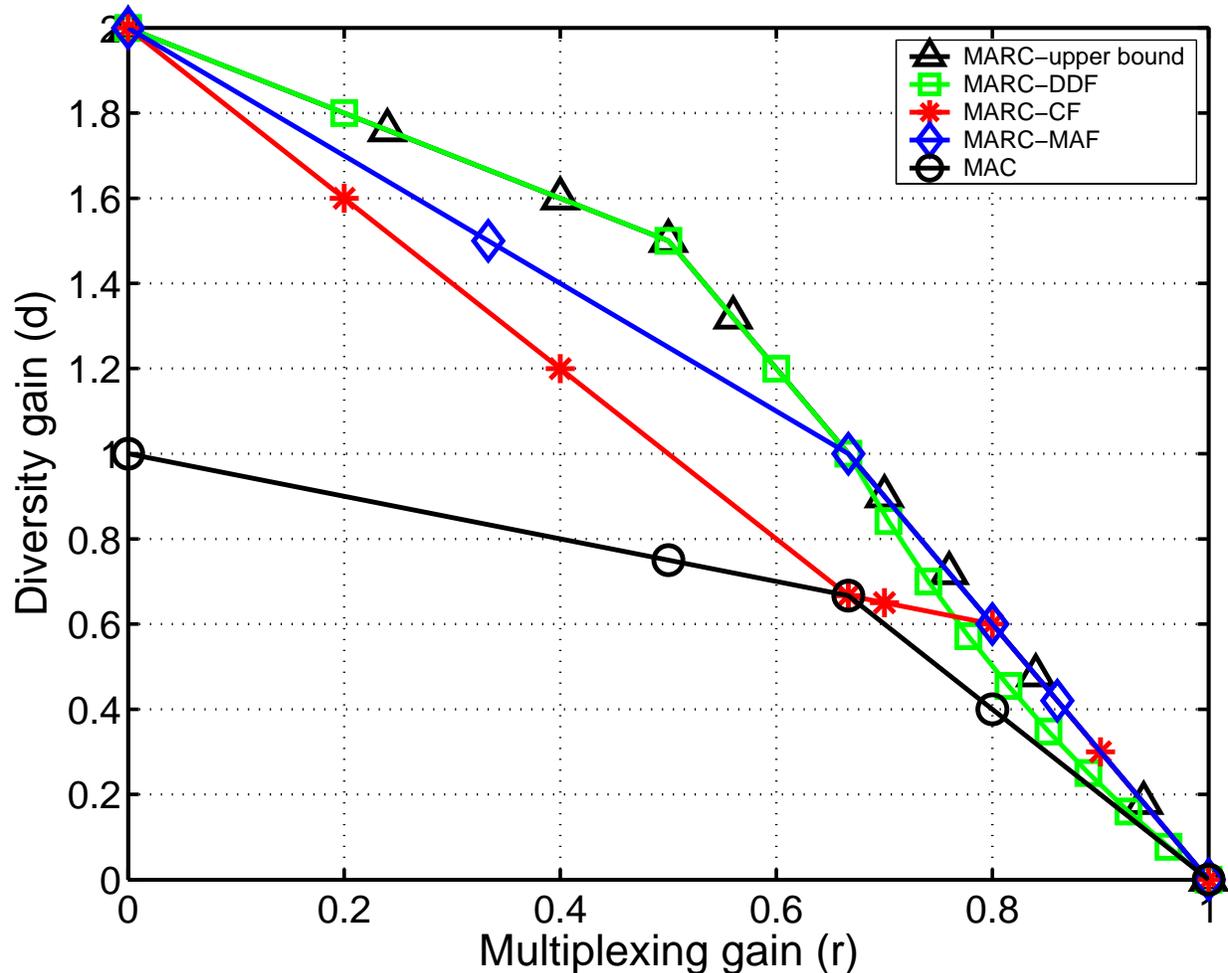


Fig. 2. DMT of the different MARC protocols.

- 5) Even over the range of multiplexing gains for which MAF becomes suboptimal, *i.e.*,  $0 \leq r \leq 2/3$ , the achieved DMT is identical to that of the NAF relay [3] with a single user. In other words, for low multiplexing gains, each user benefits from the relay as if it is the only user present. Also, in this regime, the DMT gap between DDF and MAF is much smaller compared to the gap between DDF and CF.
- 6) For all multiplexing gains, MAF outperforms MAC in terms of DMT. This observation reveals the significant advantage that multiple users can potentially gain from a single MAF relay. The DMT of DDF approaches that of the MAC in the high multiplexing regime. Thus, the gain of a complicated DDF relay diminishes for high multiplexing gains.

This section is concluded by some simulation results for CF, MAF and DDF. For CF, it is assumed that the relay listens to the source for the first half of the block, and then re-encodes and transmits the messages in the second half of the block. This assumption may not be optimum in terms of minimizing the outage probability of CF, but it does not affect the DMT of the protocol [6]. For MAF, the amplification coefficient  $b$  is chosen such that (4) holds with equality. This choice of amplification coefficient may not minimize the outage probability of MAF as well. For a multiplexing gain of  $r$ , the total data rate  $R$  is chosen to be  $r \log(1 + \rho)$ . Fig. 3 and Fig. 4 show the outage probabilities  $P_{\mathcal{O}}(R)$  of CF, DDF and MAF for  $r = 0.4$  and  $0.8$ , respectively. As can be seen in Fig. 3, for  $r = 0.4$ , the outage probability curve of DDF enjoys a steeper slope compared to that of MAF and CF, indicating a higher diversity gain for DDF. Fig. 3 also shows that for  $r = 0.4$ , MAF offers a better diversity gain relative to CF. For  $r = 0.8$ , the intersection between the curve of MAF and that of DDF in Fig. 4 suggests that MAF has a higher diversity gain. These observations are in line with the DMT analysis, *i.e.*, the diversity gain of MAF is higher than that of DDF at high multiplexing gains (*e.g.*,  $r = 0.8$ ), but smaller at low multiplexing gains (*e.g.*,  $r = 0.4$ ). It is interesting to notice that even though Fig. 2 suggests both MAF and CF provide a better DMT than DDF for  $4/5 \leq r \leq 1$ , DDF yields a better outage probability than both MAF and CF over a large range of SNRs as can be seen from Fig. 4. Furthermore, Fig. 4 demonstrates outage performance differences between MAF and CF, despite the fact that their DMT curves are identical for  $4/5 \leq r \leq 1$ . In particular, the slopes of the outage curves for CF and MAF appear to differ slightly. This small difference could be due to the finite SNR at which these simulations have been carried out, and the suboptimum choice of the amplification coefficient  $b$  in MAF. These *discrepancies* between the outage simulation results and the DMT results suggest that the complete system design requires characterization of not only the DMT, which captures the *exponential behavior* of the error probability with SNR, but also the multiplicative coefficient that captures the *geometric dependence* and the *coding gain* of the relaying protocols.

## V. CONCLUSION

In contrast to the majority of previous works on multi-access relay channel (MARC), which focus on protocols that require complicated signal processing at the relay [5], [6], this paper proposes a linear relaying protocol, *i.e.*, multi-access amplify-forward (MAF), which not only

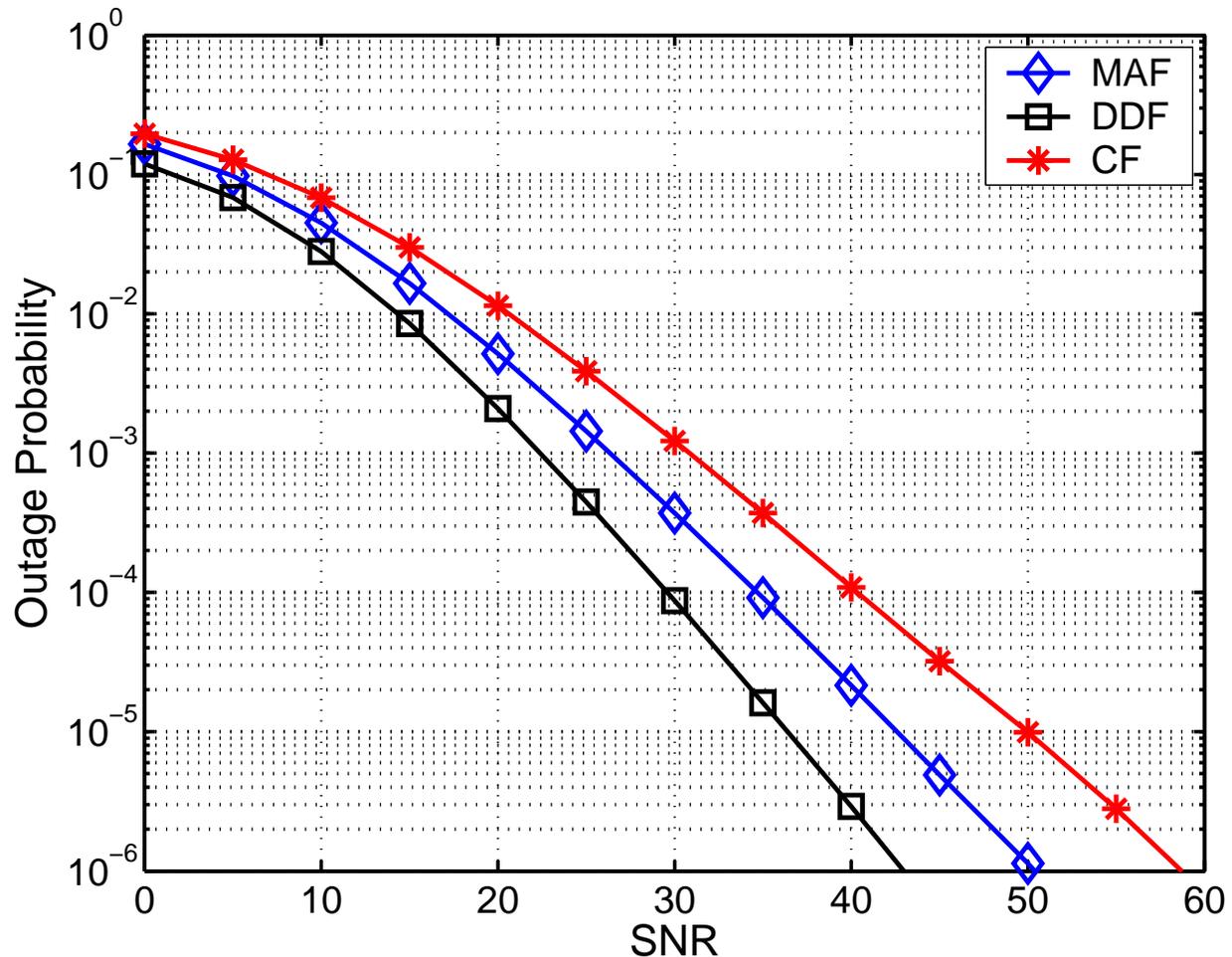


Fig. 3. Outage probabilities  $P_{\mathcal{O}}(R)$  for CF, DDF and MAF. The multiplexing gain is  $r = 0.4$ . Note that  $R = r \log(1 + \rho)$

enjoys low complexity at the relay, but also exhibits very good performance in slow-fading environments. In particular, MAF achieves the optimal diversity-multiplexing trade-off over the range  $2/3 \leq r \leq 1$ . Interestingly, for  $2/3 \leq r \leq 4/5$  neither DDF nor CF, despite their complicated relaying algorithms, is optimal. In the low multiplexing gain regime, *i.e.*,  $0 \leq r \leq 2/3$ , MAF allows each user to gain cooperative diversity as if there is no interference from the other users and no contention for the relay.

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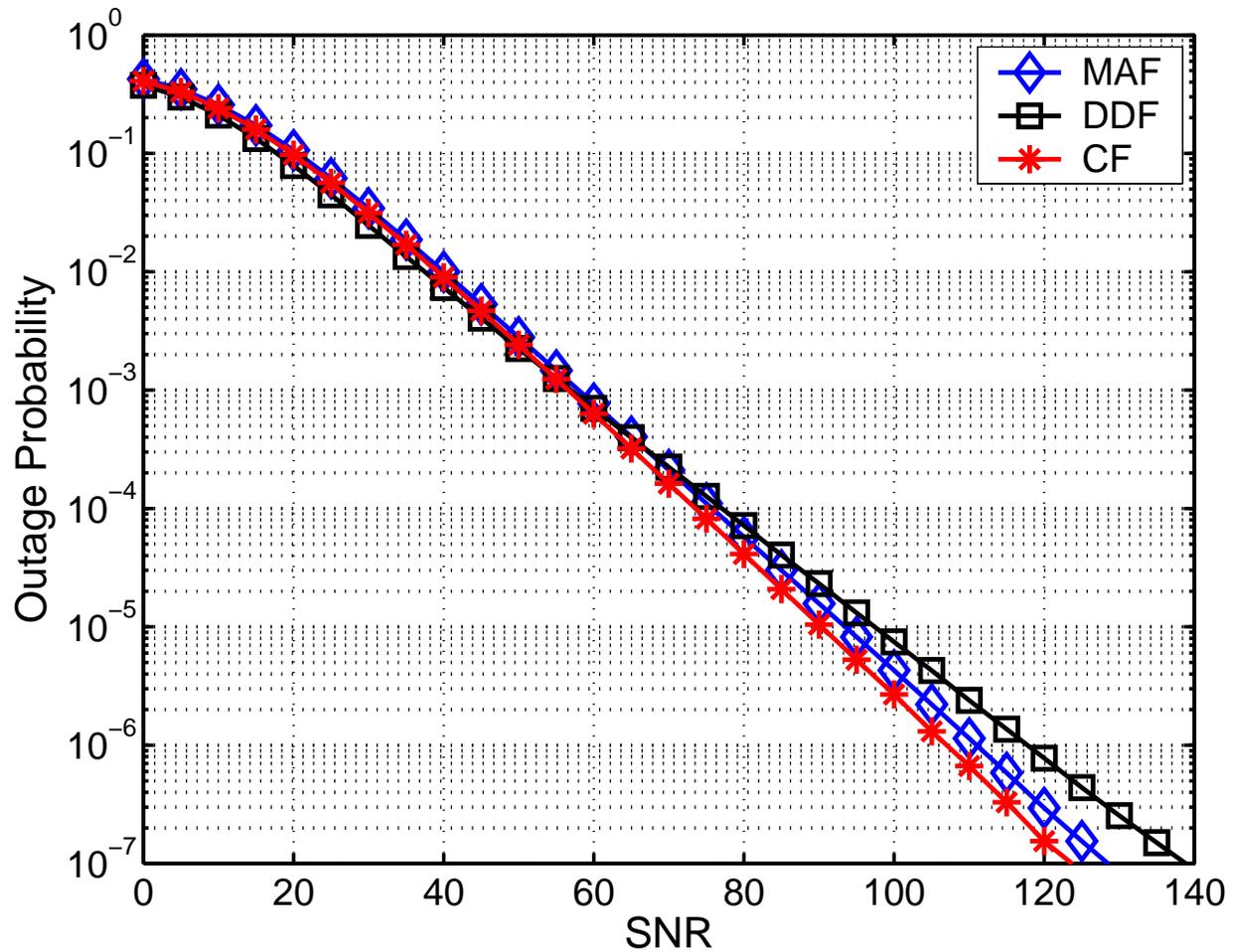


Fig. 4. Outage probabilities  $P_{\mathcal{O}}(R)$  for CF, DDF and MAF. The multiplexing gain is  $r = 0.8$ . Note that  $R = r \log(1 + \rho)$

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