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1. Show by direct calculation, using Eqs.(1.20-1.24), that

$$L_+L_- + L_z^2 - L_z = L^2$$
$$L_-L_+ + L_z^2 + L_z = L^2$$

Mathematica statements:

(a) g = $\psi(\theta, \phi)$ (define a general function)

(b) lsq = Simplify[- (Csc[θ] D[Sin[θ] D[g,θ], θ] + (Csc[θ])ˆ2 D[D[g, ϕ], ϕ])] (evaluate $L^2\psi$)

(c) lz = I D[g, ϕ] (evaluate $L_z\psi$)

(d) lz2 = I D[lz, ϕ] (evaluate $L_z^2\psi$)

(e) lpg = Eˆ(I ϕ) (D[g, θ] + I Cot[θ] D[g, ϕ]) (evaluate $L_+\psi$)

(f) lmp = Simplify[Eˆ(-I ϕ) (-D[lpg, θ] + I Cot[θ] D[lpg, ϕ])] (evaluate $L_-L_+\psi$)

(g) form1 = Simplify[lmp + lz2 - lz] (evaluate l.h.s of 1st identity)

(h) form1 === lsq (the answer to this question is "True")

(i) lmg = Eˆ(-I ϕ) (- D[g, θ] + I Cot[θ] D[g, ϕ])

(j) lpm = Simplify[Eˆ(I ϕ) (D[lmg, θ] + I Cot[θ] D[lmg, ϕ])]

(k) form2 = Simplify[lpm + lz2 + lz]

(l) form2 === lsq (the answer to this is again "True")

2. Write a Maple or Mathematica routine to obtain formulas for $\Theta_{l,m}(\theta)$ for $l = 4$ and $m \leq l$ using Eq.(1.36). With the aid of your results, give explicit formulas for $Y_{4,m}(\theta, \phi)$, $m = -4 \cdots 4$. Verify by direct calculation that $Y_{4,m}(\theta, \phi)$ are properly normalized.

Mathematica statements:

(a) l = 4

(b) Table[(-1)ˆl/l!/2ˆl Sqrt[(2 l + 1)(l + m)!/(2 (l - m)!)](1/(Sin[θ])ˆm) D[(1 - xˆ2)ˆl, {x, l - m}], {m, 0, l}]

(c) posf = FullSimplify[% /. x → Cos[θ]] (Create a table of $\Theta_{l,m}$ for nonnegative values of m.)

(d) tab1 = Table[(-1)ˆ((3 m + Abs[m])/2) Sqrt[1/(2 Pi)] Eˆ(I m ϕ) FullSimplify[posf[[Abs[m] + 1]]], {m, -l, l}] (Create a table of Y_{lm}, $m = -l \ \cdots \ l$)

(e) Table[Integrate[Sin[θ] Integrate[Conjugate[tab1[[i]]] tab1[[j]], {ϕ, 0, 2 Pi}], {θ, 0, Pi}], {i, 1, 2 l + 1}, {j, 1, 2 l + 1}] (Check orthonormality)