1. Show in detail that Eq.(4.50) follows from Eq.(4.47).

Eq.(4.50) is

\[ E^{(1)}_{ab,LS} = \eta^2 \sum_{m',\mu'} \frac{I_a m_a}{1/2 \mu_a} - \frac{I_a m'_a}{1/2 \mu'_a} - \frac{I_b m_b}{1/2 \mu_b} - \frac{I_b m'_b}{1/2 \mu'_b} \]

\[ \left[ g_{a'b'ab} \delta \mu'_a \mu_a \delta \mu'_b \mu_b - g_{a'ba'b} \delta \mu'_a \mu_a \delta \mu'_b \mu_b - (\delta \mu'_a \delta \mu'_b - \delta \mu'_b \delta \mu'_a) (U_{aa} + U_{bb}) \right] . \]

where

\[ g_{abcd} = \sum_k \frac{k}{l_a m_a} + \frac{k}{l_b m_b} + X_k(abcd) . \]

The sums over \( \mu \)'s associated with \( \delta \mu'_a \mu_a \) gives

\[ - \frac{1/2 \mu_a}{1/2 \mu_b} - \frac{1/2 \mu_a}{1/2 \mu_b} = \frac{SM_a}{SM_a} = 1 \]

Similarly, the sums over \( \mu \)'s associated with \( \delta \mu'_a \mu_a \) gives

\[ - \frac{1/2 \mu_a}{1/2 \mu_b} - \frac{1/2 \mu_a}{1/2 \mu_b} = \frac{SM_a}{SM_a} = \frac{1}{(-1)^{1/2+1/2+S}} = (-1)^{1+S} \]

The sum over \( m \)'s associated with the first two-body term can be rotated into the following form where a sum over \( M_L \) has been done and the factor \([L]\) correspondingly dropped.

\[ (-1)^{L_a + L_b + L + k} \left\{ \begin{array}{c} l_a \\ (L) \\ l_b \end{array} \right\} \]

Similarly, the sum over \( m \)'s associated with the second two-body term can be put in the form

\[ (-1)^{L_a + L_b + k} \left\{ \begin{array}{c} l_a \\ (L) \\ l_b \end{array} \right\} \]
Finally, the sums over $m$ associated with the one-body terms can be treated precisely as the sums over $\mu$ above, leading to the following result for the one-body part

$$-\eta^2 \left( 1 + (-1)^{L+S} \delta_{ab} \right) (U_{aa} + U_{bb}) = -U_{aa} - U_{bb}$$

Putting all of the above together, leads to the following expression for the first-order energy:

$$E^{(1)}_{ab,LS} = \eta^2 \sum_k \left[ (-1)^{L+k+l_a+l_b} \begin{pmatrix} l_a & l_b & L & k \\ l_a & l_b & L & k \end{pmatrix} X_k(abab) \\ + (-1)^{S+k+l_a+l_b} \begin{pmatrix} l_a & l_b & L & k \\ l_a & l_b & L & k \end{pmatrix} X_k(abba) \right] - U_{aa} - U_{bb}. $$