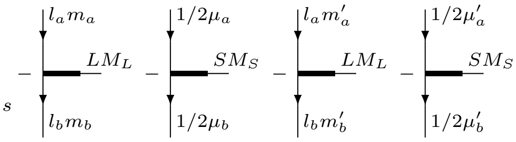
1. Show in detail that Eq.(4.50) follows from Eq.(4.47).

   Eq.(4.50) is

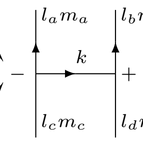
$$E^{(1)}_{ab,LS} = \eta^2 \sum_{m's\mu's}$$



$$\left[g_{a'b'ab}\delta_{\mu'_a\mu_a}\delta_{\mu'_b\mu_b} - g_{a'b'ba}\delta_{\mu'_a\mu_b}\delta_{\mu'_b\mu_a} - (\delta_{a'a}\delta_{b'b} - \delta_{a'b}\delta_{b'a})(U_{aa} + U_{bb}) \right] .$$

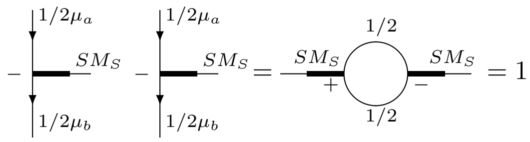
where

$$g_{abcd} = \sum_k$$

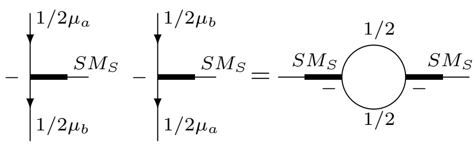


$$X_k(abcd) .$$

The sums over μ's associated with $\delta_{\mu'_a\mu_a}\delta_{\mu'_b\mu_b}$ gives

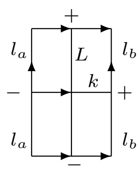


Similarly, the sums over μ's associated with $\delta_{\mu'_a\mu_b}\delta_{\mu'_b\mu_a}$ gives



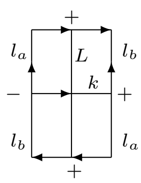
$$= (-1)^{1/2+1/2+S} = (-1)^{1+S}$$

The sum over m's associated with the first two-body term can be rotated into the following form where a sum over M_L has been done and the factor $[L]$ correspondingly dropped.



$$= (-1)^{l_a+l_b+L+k} \left\{ \begin{array}{ccc} l_a & l_b & L \\ l_b & l_a & k \end{array} \right\}$$

Similarly, the sum over m's associated with the second two-body term can be put in the form



$$= (-1)^{l_a+l_b+k} \left\{ \begin{array}{ccc} l_a & l_b & L \\ l_a & l_b & k \end{array} \right\}$$

Finally, the sums over m associated with the one-body terms can be treated precisely as the sums over μ above, leading to the following result for the one-body part

$$-\eta^2 \left(1 + (-1)^{L+S}\delta_{ab}\right)(U_{aa} + U_{bb}) = -U_{aa} - U_{bb}$$

Putting all of the above together, leads to the following expression for the first-order energy:

$$E^{(1)}_{ab,LS} = \eta^2 \sum_k \left[(-1)^{L+k+l_a+l_b} \left\{ \begin{array}{ccc} l_a & l_b & L \\ l_b & l_a & k \end{array} \right\} X_k(abab) \right.$$
$$\left. + (-1)^{S+k+l_a+l_b} \left\{ \begin{array}{ccc} l_a & l_b & L \\ l_a & l_b & k \end{array} \right\} X_k(abba) \right] - U_{aa} - U_{bb}\,.$$