1. Prove:
   
   (a) \[ \langle 0_c \bigg| \sum_{ij} (\Delta V)_{ij} : a_i^\dagger a_j : a_m^\dagger a_a : 0_c \rangle = (\Delta V)_{am} \]

   Since the fully contracted contribution to \( :a_i^\dagger a_j : a_m^\dagger a_a : \) is \( \delta_{jm} \delta_{ia} \), the sum reduces to \( (\Delta V)_{am} \).

   (b) \[ \langle 0_c \bigg| \frac{1}{2} \sum_{ijkl} g_{ijkl} : a_i^\dagger a_j^\dagger a_k : a_m^\dagger a_n : a_b a_a : 0_c \rangle = \tilde{g}_{abmn} \]

   Here, the fully contracted contribution to \( :a_i^\dagger a_j^\dagger a_k : a_m^\dagger a_n : a_b a_a : \) is \( (\delta_{ia} \delta_{jb} - \delta_{ib} \delta_{ja}) \delta_{km} \delta_{ln} - \delta_{kn} \delta_{lm} \). Therefore, the sum reduces to \( \tilde{g}_{abmn} \).

2. Prove:
   
   (a) \[ \sum_{\sigma_a \sigma_b \sigma_m \sigma_n} g_{abmn} g_{mnab} = 2 \sum_{l,k} \left\{ \begin{array}{c} l_a \\ l_b \\ l_m \\ l_k \end{array} \right\} X_k(abnm)X_l(mnab) \]

   The sums over magnetic quantum numbers are carried out graphically as follows:

   \[ m = (-1)^{a+b+m+n} \]

   The spin sum in this case is 2 and the phase accounts for the rearrangement of arguments in \( X_k(abnm) \)

   (b) \[ \sum_{\sigma_a \sigma_b \sigma_m \sigma_n} g_{abmn} g_{mnab} = 4 \sum_{l} \frac{1}{[l]} X_l(mnab)X_l(mnab) \]

   This is an essentially trivial case. The sum over magnetic quantum numbers is \( (-1)^{a+b+m+n} \delta_{kl}/[l] \) and the spin sum is 4, leading to the desired result.