1. Prove:

   (a)
$$\left\langle 0_c \left| \sum_{ij} (\Delta V)_{ij} :a_i^\dagger a_j::a_m^\dagger a_a: \right| 0_c \right\rangle = (\Delta V)_{am}$$

   Since the fully contracted contribution to $:a_i^\dagger a_j::a_m^\dagger a_a:$ is $\delta_{jm}\delta_{ia}$, the sum reduces to $(\Delta V)_{am}$.

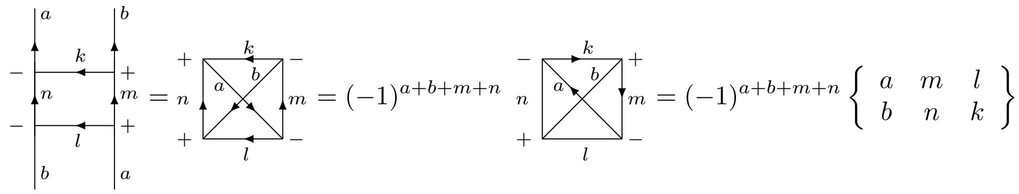
   (b)
$$\left\langle 0_c \left| \frac{1}{2} \sum_{ijkl} g_{ijkl} :a_i^\dagger a_j^\dagger a_l a_k::a_m^\dagger a_n^\dagger a_b a_a: \right| 0_c \right\rangle = \tilde{g}_{abmn}$$

   Here, the fully contracted contribution to $:a_i^\dagger a_j^\dagger a_l a_k::a_m^\dagger a_n^\dagger a_b a_a:$ is $(\delta_{ia}\delta_{jb} - \delta_{ib}\delta_{ja})(\delta_{km}\delta_{ln} - \delta_{kn}\delta_{lm})$. Therefore, the sum reduces to $\tilde{g}_{abmn}$

2. Prove:

   (a)
$$\sum_{\substack{m_a m_b m_m m_n \\ \sigma_a \sigma_b \sigma_m \sigma_n}} g_{abnm}\; g_{mnab} = 2 \sum_{l,k} \left\{ \begin{array}{ccc} l_a & l_m & l \\ l_b & l_n & k \end{array} \right\} X_k(nmab) X_l(mnab)$$

   The sums over magnetic quantum numbers are carried out graphically as follows:



   The spin sum in this case is 2 and the phase accounts for the rearrangement of arguments in $X_k(abnm)$

   (b)
$$\sum_{\substack{m_a m_b m_m m_n \\ \sigma_a \sigma_b \sigma_m \sigma_n}} g_{abmn}\; g_{mnab} = 4 \sum_{l} \frac{1}{[l]} X_l(mnab) X_l(mnab)$$

   This is an essentially trivial case. The sum over magnetic quantum numbers is $(-1)^{a+b+m+n}\delta_{kl}/[l]$ and the spin sum is 4, leading to the desired result.