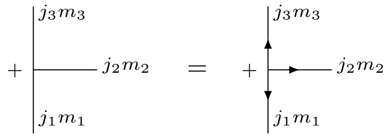
Atomic Physics I 80301 Exam I Sept. 26, 2005

Work any three of the following four problems.

1. Derive the identity



2. Show that the spherical harmonics $Y_{kq}(\theta, \phi)$, $q = -k, -k+1, \cdots, k$ are components of a spherical tensor operator of rank k.

$C(l, 1/2, j; m - m_s, m_s, m)$		
	$m_s = 1/2$	$m_s = -1/2$
$j = l + 1/2$	$\sqrt{\frac{l+m+1/2}{2l+1}}$	$\sqrt{\frac{l-m+1/2}{2l+1}}$
$j = l - 1/2$	$-\sqrt{\frac{l-m+1/2}{2l+1}}$	$\sqrt{\frac{l+m+1/2}{2l+1}}$

3. Consider the 12-fold degenerate set of product wave functions:

$$\psi_{2p,m,\sigma}(\boldsymbol{r}_1)\psi_{1s,\mu}(\boldsymbol{r}_2) = \frac{1}{r_1 r_2} P_{2p}(r_1) P_{1s}(r_2) Y_{1m}(\hat{r}_1) Y_{00}(\hat{r}_2) \chi_\sigma(1)\chi_\mu(2).$$

   (a) Combine these wave functions to give eigenstates of L^2, L_z, S^2, S_z, where $\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2$ and $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$.

   (b) With the aid of the above result, write down all possible *antisymmetric* angular momentum eigenstates describing $1s2p$ levels of helium. What is the number of such states?

4. Show that the ionization energy of an atom with one valence electron is $-\epsilon_v$ in the "frozen-core" Hartree-Fock approximation. (Here, ϵ_v is the eigenvalue of the valence electron Hartree-Fock equation.)