First, express the radial wave functions P and Q in terms of F1 and F2. In the equations below, we set $\frac{E}{c^2}=\epsilon$ for simplicity.

$\boldsymbol{x = 2\lambda r}$

$2r\lambda$

$\boldsymbol{P[\mathrm{x\_}]\text{:=}\mathrm{Sqrt}[1+\epsilon]\mathrm{Exp}[-x/2](\mathrm{F1}[x]+\mathrm{F2}[x])}$

$\boldsymbol{Q[\mathrm{x\_}]\text{:=}\mathrm{Sqrt}[1-\epsilon]\mathrm{Exp}[-x/2](\mathrm{F1}[x]-\mathrm{F2}[x])}$

Next, write down the radial Dirac equations

$\boldsymbol{\mathrm{eq1} = (-Z/r + c\text{^}2)P[x] + c(D[Q[x],r] - \kappa/rQ[x]) == c\text{^}2\epsilon P[x]}$

$$e^{-r\lambda}\left(c^2-\frac{Z}{r}\right)\sqrt{1+\epsilon}(\mathrm{F1}[2r\lambda]+\mathrm{F2}[2r\lambda]) + c\left(-\frac{e^{-r\lambda}\sqrt{1-\epsilon}\kappa(\mathrm{F1}[2r\lambda]-\mathrm{F2}[2r\lambda])}{r} - e^{-r\lambda}\sqrt{1-\epsilon}\lambda(\mathrm{F1}[2r\lambda]-\mathrm{F2}[2r\lambda]) + e^{-r\lambda}\sqrt{1-\epsilon}\left(2\lambda\mathrm{F1}'[2r\lambda]-2\lambda\mathrm{F2}'[2r\lambda]\right)\right) == c^2e^{-r\lambda}\epsilon\sqrt{1+\epsilon}(\mathrm{F1}[2r\lambda]+\mathrm{F2}[2r\lambda])$$

$\boldsymbol{\mathrm{eq2} = (-Z/r - c\text{^}2)Q[x] - c(D[P[x],r] + \kappa/rP[x]) == c\text{^}2\epsilon Q[x]}$

$$e^{-r\lambda}\left(-c^2-\frac{Z}{r}\right)\sqrt{1-\epsilon}(\mathrm{F1}[2r\lambda]-\mathrm{F2}[2r\lambda]) - c\left(\frac{e^{-r\lambda}\sqrt{1+\epsilon}\kappa(\mathrm{F1}[2r\lambda]+\mathrm{F2}[2r\lambda])}{r} - e^{-r\lambda}\sqrt{1+\epsilon}\lambda(\mathrm{F1}[2r\lambda]+\mathrm{F2}[2r\lambda]) + e^{-r\lambda}\sqrt{1+\epsilon}\left(2\lambda\mathrm{F1}'[2r\lambda]+2\lambda\mathrm{F2}'[2r\lambda]\right)\right) == c^2e^{-r\lambda}\sqrt{1-\epsilon}\epsilon(\mathrm{F1}[2r\lambda]-\mathrm{F2}[2r\lambda])$$

Now solve the equations for dF1/dx and dF2/dx

$\boldsymbol{\mathrm{sol} = \mathrm{Solve}[\{\mathrm{eq1},\mathrm{eq2}\},\{\mathrm{F1}'[2\lambda r],\mathrm{F2}'[2\lambda r]\}]}$

$$\Big\{\Big\{\mathrm{F1}'[2r\lambda]\to -\frac{1}{2cr\sqrt{1-\epsilon}\lambda}\Big(c^2r\sqrt{1+\epsilon}\mathrm{F1}[2r\lambda] - Z\sqrt{1+\epsilon}\mathrm{F1}[2r\lambda] - c^2r\epsilon\sqrt{1+\epsilon}\mathrm{F1}[2r\lambda] - c\sqrt{1-\epsilon}\kappa\mathrm{F1}[2r\lambda] - cr\sqrt{1-\epsilon}\lambda\mathrm{F1}[2r\lambda] + c^2r\sqrt{1+\epsilon}\mathrm{F2}[2r\lambda] - Z\sqrt{1+\epsilon}\mathrm{F2}[2r\lambda] - c^2r\epsilon\sqrt{1+\epsilon}\mathrm{F2}[2r\lambda] + c\sqrt{1-\epsilon}\kappa\mathrm{F2}[2r\lambda] + cr\sqrt{1-\epsilon}\lambda\mathrm{F2}[2r\lambda]\Big) - \Big(Z\mathrm{F1}[2r\lambda] + c\sqrt{1-\epsilon}\sqrt{1+\epsilon}\kappa\mathrm{F1}[2r\lambda] - c^2r\mathrm{F2}[2r\lambda] + Z\epsilon\mathrm{F2}[2r\lambda] + c^2r\epsilon^2\mathrm{F2}[2r\lambda] - cr\sqrt{1-\epsilon}\sqrt{1+\epsilon}\lambda\mathrm{F2}[2r\lambda]\Big)/\Big(2cr\sqrt{1-\epsilon}\sqrt{1+\epsilon}\lambda\Big),$$

$$\mathrm{F2}'[2r\lambda]\to -\Big(Z\mathrm{F1}[2r\lambda] + c\sqrt{1-\epsilon}\sqrt{1+\epsilon}\kappa\mathrm{F1}[2r\lambda] - c^2r\mathrm{F2}[2r\lambda] + Z\epsilon\mathrm{F2}[2r\lambda] + c^2r\epsilon^2\mathrm{F2}[2r\lambda] - cr\sqrt{1-\epsilon}\sqrt{1+\epsilon}\lambda\mathrm{F2}[2r\lambda]\Big)/\Big(2cr\sqrt{1-\epsilon}\sqrt{1+\epsilon}\lambda\Big)\Big\}\Big\}$$

Simplify the result; dF1/dx -> f1p dF2/dx -> f2p

$\boldsymbol{\mathrm{f1p} = \mathrm{Expand}[\mathrm{FullSimplify}[\mathrm{F1}'[2\lambda r]/.\mathrm{sol}]]}$

$$\left\{\frac{1}{2}\mathrm{F1}[2r\lambda] - \frac{c\mathrm{F1}[2r\lambda]}{2\sqrt{1-\epsilon^2}\lambda} + \frac{Z\epsilon\mathrm{F1}[2r\lambda]}{2cr\sqrt{1-\epsilon^2}\lambda} + \frac{c\epsilon^2\mathrm{F1}[2r\lambda]}{2\sqrt{1-\epsilon^2}\lambda} + \frac{Z\mathrm{F2}[2r\lambda]}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{\kappa\mathrm{F2}[2r\lambda]}{2r\lambda}\right\}$$

$\boldsymbol{\mathrm{f2p} = \mathrm{Expand}[\mathrm{FullSimplify}[\mathrm{F2}'[2\lambda r]/.\mathrm{sol}]]}$

$$\Big\{-\frac{Z\mathrm{F1}[2r\lambda]}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{\kappa\mathrm{F1}[2r\lambda]}{2r\lambda} + \frac{1}{2}\mathrm{F2}[2r\lambda] +$$

$\frac{c\text{F2}[2r\lambda]}{2\sqrt{1-\epsilon^2}\lambda} - \frac{Z\epsilon\text{F2}[2r\lambda]}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{c\epsilon^2\text{F2}[2r\lambda]}{2\sqrt{1-\epsilon^2}\lambda}\Big\}$

Find coefficients of F1 and F2 on RHS of equations

f11 = Coefficient[f1p, F1[2λr]]

$\left\{\frac{1}{2} - \frac{c}{2\sqrt{1-\epsilon^2}\lambda} + \frac{Z\epsilon}{2cr\sqrt{1-\epsilon^2}\lambda} + \frac{c\epsilon^2}{2\sqrt{1-\epsilon^2}\lambda}\right\}$

f12 = Coefficient[f1p, F2[2λr]]

$\left\{\frac{Z}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{\kappa}{2r\lambda}\right\}$

f21 = Coefficient[f2p, F1[2λr]]

$\left\{-\frac{Z}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{\kappa}{2r\lambda}\right\}$

f22 = Coefficient[f2p, F2[2λr]]

$\left\{\frac{1}{2} + \frac{c}{2\sqrt{1-\epsilon^2}\lambda} - \frac{Z\epsilon}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{c\epsilon^2}{2\sqrt{1-\epsilon^2}\lambda}\right\}$

Combine 1st, 2nd, and 4th terms of f11 and of f22 to further simplify

Simplify[f11[[1, 1]] + f11[[1, 2]] + f11[[1, 4]]/.
λ->cSqrt[1 − ϵ^2]]

0

Simplify[f22[[1, 1]] + f22[[1, 2]] + f22[[1, 4]]/.
λ->cSqrt[1 − ϵ^2]]

1

Recombine the simplified coefficients and write out the resulting RHS terms

g11 = f11[[1, 3]]F1[x]

$\frac{Z\epsilon\text{F1}[2r\lambda]}{2cr\sqrt{1-\epsilon^2}\lambda}$

g12 = f12[[1]]F2[x]

$\left(\frac{Z}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{\kappa}{2r\lambda}\right)\text{F2}[2r\lambda]$

g21 = f21[[1]]F1[x]

$\left(-\frac{Z}{2cr\sqrt{1-\epsilon^2}\lambda} - \frac{\kappa}{2r\lambda}\right)\text{F1}[2r\lambda]$

g22 = (1 + f22[[1, 3]])F2[x]

$\left(1 - \frac{Z\epsilon}{2cr\sqrt{1-\epsilon^2}\lambda}\right)\text{F2}[2r\lambda]$

Clear[x]

r = x/(2λ)

$\frac{x}{2\lambda}$

g11 + g12

$\frac{Z\epsilon\text{F1}[x]}{cx\sqrt{1-\epsilon^2}} + \left(\frac{Z}{cx\sqrt{1-\epsilon^2}} - \frac{\kappa}{x}\right)\text{F2}[x]$

g21 + g22

$\left(-\frac{Z}{cx\sqrt{1-\epsilon^2}} - \frac{\kappa}{x}\right)\text{F1}[x] + \left(1 - \frac{Z\epsilon}{cx\sqrt{1-\epsilon^2}}\right)\text{F2}[x]$

Therefore, noting that $\lambda = \text{c}\ \sqrt{1-\epsilon^2}$, we find

$\frac{\text{dF1}}{\text{dx}} =$ `g11 + g12` $= \frac{Z\epsilon}{\lambda x}$ `F1[x]` $+ (\frac{Z}{\lambda x} - \frac{\kappa}{x})$ F2[x]

$\frac{\text{dF2}}{\text{dx}} =$ `g21 + g22` $= (-\frac{Z}{\lambda \text{x}} - \frac{\kappa}{x})$ F1[x]+ $(1 - \frac{Z\epsilon}{\lambda x})$ F2[x]