Your Answers to these problems will be collected in class on Sept. 14.

Use the attached Maple routine "coul(n,l)" or write an equivalent Mathematica routine to evaluate radial Coulomb wave functions.

1. Plot the radial functions $P_{nl}(r)$ for $n = 4$ and all possible values of l.

2. Show by direct calculation that the $l = 1$ radial functions $P_{nl}(r)$ satisfy

$$\int_0^\infty dr P_{nl}(r) P_{n'l}(r) = \delta_{nn'}$$

3. The Coulomb field of the nucleus is modified at short distances by the finite size of the nuclear charge distribution.

   (a) Assuming that the nuclear charge density is constant, show that the potential inside the nucleus is

$$V = -\frac{Z}{R}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right], \quad r \leq R$$

   (b) Show by explicit calculation for n from 2 to 5 that the leading term in powers of R of the finite size shift in energy is

$$\Delta E_{n,0} = \frac{2Z^2}{n^3}(ZR)^2 \quad \text{for } l = 0$$

$$\Delta E_{n,1} = \frac{(n^2-1)Z^2}{9n^5}(ZR)^4 \quad \text{for } l = 1$$

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coul := proc(n,l)
local lam,an,x,p;
lam := Z/n;
an := sqrt(Z*(n+l)!/(n-l-1)!)/(n*(2*l+1)!);
x := 2*lam*r;
p := an*x^(l+1)*exp(-x/2)*chg(-n+l+1,2*l+2,x);
end:

chg := proc(n,b,x)
local aa,bb,dd,tt,ff;
if n=0 then RETURN(1)
elif n<0 then
  aa := n;
  bb := b;
  dd := 1;
  tt := 1;
  ff := 1;
  for i from 1 to -n do
     tt := tt*aa*x/(dd*bb);
     aa := aa+1;
     bb := bb+1;
     dd := dd+1;
     ff := ff+tt;
  od;
  RETURN(ff);
else
  RETURN(0);
fi;
end:
```