The answer to these problems will be collected in class on Nov. 2.

1. **Zeeman effect:** The vector potential for a uniform magnetic $\boldsymbol{B}$ can be written

$$\boldsymbol{A} = \frac{1}{2}[\boldsymbol{B} \times \boldsymbol{r}]$$

(a) Show that the interaction Hamiltonian of an electron with this field is

$$h_{\rm int}(r) = \frac{iec}{2} B\sqrt{2}\, r \left(\boldsymbol{\alpha} \cdot \boldsymbol{C}^{(0)}_{10}\right),$$

assuming that the field is oriented along the z axis.

(b) Show that the expectation value of the *many-electron* Hamiltonian $H_{\rm int} = \sum_i h_{\rm int}(r_i)$ for a one-valence electron atom in state v reduces to

$$\langle v m_v | H_{\rm int} | v m_v \rangle = -ecB\kappa \, \langle -\kappa_v m_m | C^1_0 | \kappa_v m_v \rangle \, (r)_{vv}$$

in the independent-particle approximation. Here,

$$(r)_{vv} = 2 \int_0^\infty dr\, r\, P_v(r)\, Q_v(r)$$

(c) Evaluate $(r)_{vv}$ in the Pauli approximation and show that the interaction energy can be written

$$W = -\mu_B B g_v m_v$$

where the Landé g-factor is given by

$$g_v = \frac{\kappa_v(\kappa_v - 1/2)}{j_v(j_v + 1)}.$$

This factor has the value 2, 2/3, 4/3, 4/5, 6/5, for $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, $d_{5/2}$ states, respectively. In the above, $\mu_B = e/2m$ is the Bohr magneton. Its value is $e/2$ in atomic units.

2. **Isotope Shift in Li:**

(a) Using experimental energies from the NIST data base, evaluate the normal mass shift correction to energies of the $2s$ and $2p$ states of the isotopes ^{6}Li and ^{7}Li.

(b) Assuming that the $1s$ wave function of Li is a Coulomb wave function in a field with $Z = 3 - 5/16$ and that the $2s$ and $2p$ wave functions are Coulomb wave functions in a field $Z = 1 + 1/8$, determine the specific mass shift for $2s$ and $2p$ states of ^{6}Li and ^{7}Li.

(c) Combine the above calculations to determine the difference between $2s$ energies in the two isotopes. Repeat the calculation for $2p$ levels. What shift (cm^{-1}) is expected in the $2s - 2p$ transition energy? What shift (MHz) is expected in the transition frequency? What shift (Å) is expected in the transition wavelength?