

- Angular Momentum Operators:

$$\begin{aligned}
 J_x^\dagger &= J_x, & J_y^\dagger &= J_y, & J_z^\dagger &= J_z \\
 [J_x, J_y] &= iJ_z, & [J_y, J_z] &= iJ_x, & [J_z, J_x] &= iJ_y \\
 J_\pm &= J_x \pm iJ_y \\
 [J^2, J_z] &= 0, & [J^2, J_\pm] &= 0, & [J_z, J_\pm] &= \pm J_\pm \\
 J^2|j, m\rangle &= j(j+1)|j, m\rangle, & j &= \text{integer or half integer} \\
 J_z|j, m\rangle &= m|j, m\rangle, & -j &\leq m \leq j \\
 J_\pm|j, m\rangle &= \sqrt{(j \mp m)(j \pm m + 1)}|j, m \pm 1\rangle
 \end{aligned}$$

- Examples:

1. Orbital Angular Momentum \mathbf{L} :

Operators: $L_x = \frac{1}{i}[y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}] + \text{cyclic}$.

Eigenvalues: $l = 0, 1, 2, \dots, \quad -l \leq m \leq l$

Eigenfunctions: $|l, m\rangle = Y_{lm}(\theta, \phi)$

2. Spin 1/2 Angular Momentum: Operators:

$$s_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad s_y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad s_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Eigenvalue: $s = 1/2, \mu = \pm 1/2$

Eigenfunctions:

$$\chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

3. Spin 1 Angular Momentum: Operators:

$$s_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad s_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad s_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Eigenvalue: $s = 1, \quad \mu = 1, 0, -1$

Eigenfunctions:

$$\xi_1 = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \quad \xi_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \xi_{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$