Answers to Problem Set 5 **Physics 607** **Oct. 1, 2001**

1. The radial $n = 2$ wave functions for $\kappa = \pm 1$ are:

$$
\begin{aligned}
P_{2\kappa}(r) &= \sqrt{1 + N/2}\, \mathcal{N}_{2\kappa}\; x^{\gamma_1} e^{-x/2} \left[(N-\kappa)(1 - x/b) - 1\right] \\
Q_{2\kappa}(r) &= \sqrt{1 - N/2}\, \mathcal{N}_{2\kappa}\; x^{\gamma_1} e^{-x/2} \left[(N-\kappa)(1 - x/b) + 1\right]
\end{aligned}
$$

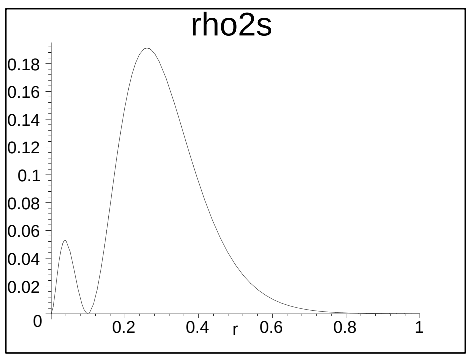
with $x = 2Zr/N$, $\mathcal{N}_{2\kappa} = \sqrt{Z\, b/[2\,\Gamma(b)\,(N-\kappa)]}$, $b = 2\gamma_1 + 1$, $N = \sqrt{2 + 2\,\gamma_1}$, $\gamma_1 = \sqrt{1 - \alpha^2 Z^2}$. The wave functions for $\kappa = -2$ are

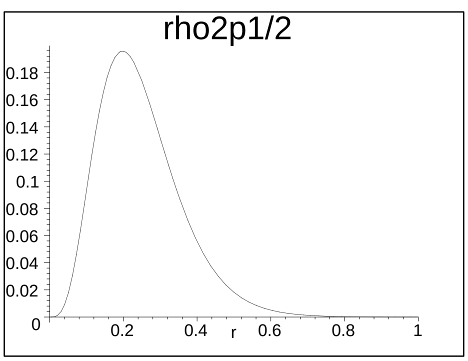
$$
\begin{aligned}
P_{2,-2}(r) &= \sqrt{1 + \gamma_2/2}\, \mathcal{N}_{2,-2}\; y^{\gamma_2} e^{-y/2} \\
Q_{2,-2}(r) &= \sqrt{1 - \gamma_2/2}\, \mathcal{N}_{2,-2}\; y^{\gamma_2} e^{-y/2},
\end{aligned}
$$

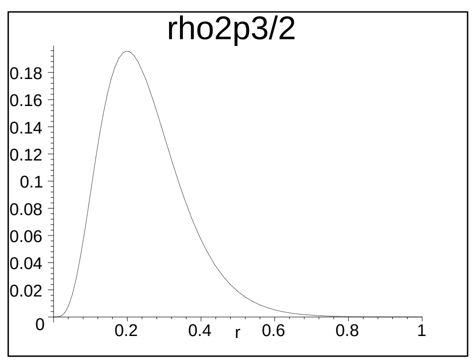
with $y = Zr$, $\mathcal{N}_{2,-2} = \sqrt{Z/[2\,\Gamma(2\gamma_2 + 1)]}$, $\gamma_2 = \sqrt{4 - \alpha^2 Z^2}$.

2. The normalization condition may be verified by carrying out integrals of the densities in the next problem numerically.

3. Plots of the radial densities are shown below:







4.

$$\gamma_1 = \sqrt{1-\alpha^2 Z^2} \qquad \gamma_2 = \sqrt{4-\alpha^2 Z^2} \qquad N = \sqrt{2+2\,\gamma_1}$$

$$\left\langle \frac{1}{r} \right\rangle_{2s} = \frac{Z}{2\,N\,\gamma_1} \qquad \left\langle \frac{1}{r} \right\rangle_{2p_{1/2}} = \frac{Z}{2\,N\,\gamma_1} \qquad \left\langle \frac{1}{r} \right\rangle_{2p_{3/2}} = \frac{Z}{2\,\gamma_2}$$

$$\langle r \rangle_{2s} = \frac{(6\,\gamma_1+5)\,N+2}{4\,Z} \qquad \langle r \rangle_{2p_{1/2}} = \frac{(6\,\gamma_1+5)\,N-2}{4\,Z} \qquad \langle r \rangle_{2p_{3/2}} = \frac{2\,\gamma_2+1}{Z}$$

5.

$$\Delta E_{n\kappa} = \frac{3Z}{\gamma(2\gamma+1)(2\gamma+3)} N_{n\kappa}^2 \left(\frac{2ZR}{N}\right)^{2\gamma} \times \left\{(N-\kappa)^2 + (n-k)^2 - 2\frac{\gamma+n-k}{N}(N-\kappa)(n-k)\right\}$$

State	H (cm^{-1})	U (eV)
$1s$	0.0000339	272.657
$2s$	0.0000042	51.995
$2p_{1/2}$	0.0	5.960
$2p_{3/2}$	0.0	0.000