Answers to Problem Set 7 **Physics 607** **Oct. 19, 2001**

1. Number of substates:

   (a) (nsn'l) in LS coupling:
   In this case, $L = l$ and there are only two possibilities ^1L and ^3L. The first has $2L+1$ substates and the second has $(2S+1)(2L+1) = 3(2L+1)$ substates. The total is $N = 4(2L+1) \equiv 8l + 4$ substates.

   (b) (nsn'l) in jj coupling:
   There are two possibilities $(ns_{1/2}n'l_{l-1/2})$ with $2\times(2(l-1/2)+1) = 4l$ substates and $(ns_{1/2}n'l_{l+1/2})$ with $2 \times (2(l+1/2)+1) = 4l+4$ substates. The total is again $N = 8L + 4$ substates.

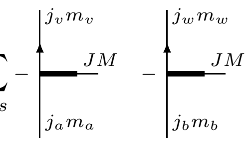
   (c) $(nd)^2$ in LS coupling:
   Since $L + S$ is even we have ^1S, ^3P, ^1D, ^3F, and ^1G. The number of states is $N = 1 + 3 \times 3 + 5 + 3 \times 7 + 9 = 45$.

   (d) $(nd)^2$ in jj coupling:
   We have the jj configurations $(nd_{3/2})^2$, $(nd_{5/2})^2$, and $(nd_{3/2}nd_{5/2})$. For $(nd_{3/2})^2$ only $J = 0$ and 2 are possible with $1 + 5 = 6$ substates; for $(nd_{5/2})^2$ only $J = 0$, 2, and 4 are possible with $1 + 5 + 9 = 15$ substates; for $(nd_{3/2}nd_{5/2})$, $J = 1$, 2, 3, and 4 are possible with $3 + 5 + 7 + 9 = 24$ substates. The total number of substates is thus $N = 6 + 15 + 24 = 45$.

2. Cases (a-d) are normally ordered and have core expectation values 0. Cases (e) and (f) are not normally ordered. The core expectation values for case (e):
$$\langle a_c^\dagger a_d^\dagger a_a a_b \rangle = \delta_{bc}\delta_{ad} - \delta_{bd}\delta_{ac},$$
and for case (f):
$$\langle a_c^\dagger a_b a_d^\dagger a_c \rangle = \delta_{bc}\delta_{dc}.$$

3. We have $|I\rangle = a_a|0_c\rangle$ and $|F\rangle = a_m^\dagger a_b a_c|0_c\rangle$. It follows that
$$\langle F|V_I|I\rangle = \frac{1}{2}\sum_{ijkl} g_{ijkl}\langle 0_c|a_c^\dagger a_b^\dagger a_m :a_i^\dagger a_j^\dagger a_l a_k: a_a|0_c\rangle$$
$$\equiv \frac{1}{2}\sum_{ijkl} g_{ijkl}\langle 0_c| :a_c^\dagger a_b^\dagger a_m::a_i^\dagger a_j^\dagger a_l a_k::a_a: |0_c\rangle$$
which, using Wick's theorem, reduces to
$$= \frac{1}{2}\sum_{ijkl} g_{ijkl}(\delta_{ia}\delta_{jm} - \delta_{im}\delta_{ja})(\delta_{kb}\delta_{lc} - \delta_{lc}\delta_{lb})$$
$$= g_{ambc} - g_{amcb} = \tilde{g}_{ambc}$$

4. The first-order energy is

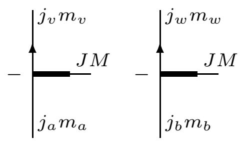
$$E^{(1)}(j_v j_w,\, J) = \sum_{m's}$$


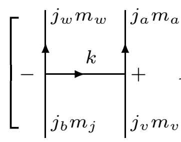
$$\langle 0_c | a_b^\dagger a_w \; V_I \; a_v^\dagger a_a | 0_c \rangle,$$

where w & v and a & b differ only in the values of the magnetic quantum numbers. With the aid of Wick's theorem, we find

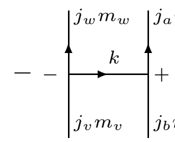
$$\langle 0_c | a_b^\dagger a_w \; V_I \; a_v^\dagger a_a | 0_c \rangle = g_{wabv} - g_{wavb} + \Delta_{wv}\delta_{ba} - \Delta_{ba}\delta_{wv}$$

The two-particle part of the first-order energy is

$$\sum_{m's}$$


$$\sum_k \Bigg[$$


$$X_k(wabv)$$

$$-$$


$$X_k(wavb) \Bigg]$$

After summing over m values, this reduces to

$$\frac{(-1)^{J+j_v-j_a}}{[J]} \left[X_J(wabv) + [J] \sum_k \left\{ \begin{array}{ccc} j_v & j_a & J \\ j_b & j_w & k \end{array} \right\} X_k(wavb) \right].$$

This leads to the answer on setting $w = v$ and $b = a$.